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METHODOLOGY OF THE ISOMETRIC POLAR CARTESIAN TRIGONOMETRIC CIRCLE IN THE GEOGEBRA SOFTWARE DEMONSTRATING THE RATIONALITY OF THE CONSTANT Π

ORIGINAL ARTICLE

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ABSTRACT

The article will present mathematical models demonstrating calculations of the relationships and proportions of the infinite periodic trigonometric circle using the GeoGebra software in the Cartesian, isometric, and polar planes. It will consider the first quadrant of the trigonometric cycle for the calculations of the trigonometric identities functions and apply the same reasoning calculations for the other quadrants. The infinite relationships and proportions between the circumferences' perimeters and their diameters, the angles (radian arcs) of the circumferences, and square roots. The oldest universal constant of mathematics, involved in circular symmetries, circular paths of stars and planets, in the propagation of electromagnetic fields, circles, and spheres and their relationships and proportions, are all approximated. Its relationship can be known exactly, or we must limit ourselves to the approximations of the calculations of the number π. This procedure can be calculated with the help of computing, and trying to obtain its rational and periodic value with a ruler and compass will only lead to frustration. The demonstration carried out in the GeoGebra software, through mathematical models in the Cartesian, isometric, and polar planes, shows the interlinked calculations and theorems of the relationships and proportions with rational and periodic values of the infinite sums of the rational periodic calculations of the number π (pi) (BECKMANN, 1971).

Keywords: Number π (pi), Trigonometric circle, Radian angles, Polygons.

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1. INTRODUCTION

Study of mathematics responsible for the relationship and proportion existing between the sides and angles of a right triangle. They have known fractional values represented for the relationships of sine, cosine, tangent, cotangent, cosecant, and secant. From the 15th century to the modernity of calculations and the creation of theoretical situations related to the study of angles in differential and integral calculus functions by scientists Isaac Newton and Leibniz, with definitive methods in the mathematical field, these are constantly employed in other sciences such as Medicine, Engineering, Physics, Chemistry, Geography, Astronomy, Biology, Cartography, and Navigation.

2. DEVELOPMENT

2.1 TRIGONOMETRIC CIRCUMFERENCE IN THE CARTESIAN PLANE

In the orthogonal Cartesian system, let's consider point A (0,1) on the A(x) axis, with an abscissa of 1. We then construct a circle with center at the origin O (0,0) of the system, passing through A, with a unit radius. We will conventionally consider point A as the origin of the oriented arcs of this circle, meaning that to traverse these arcs, point A will always be the starting point. Thus, given a plane α, a point O (0,0), at a distance Radius (r), we have: C: $x^2 + y^2$ =IR (real numbers).

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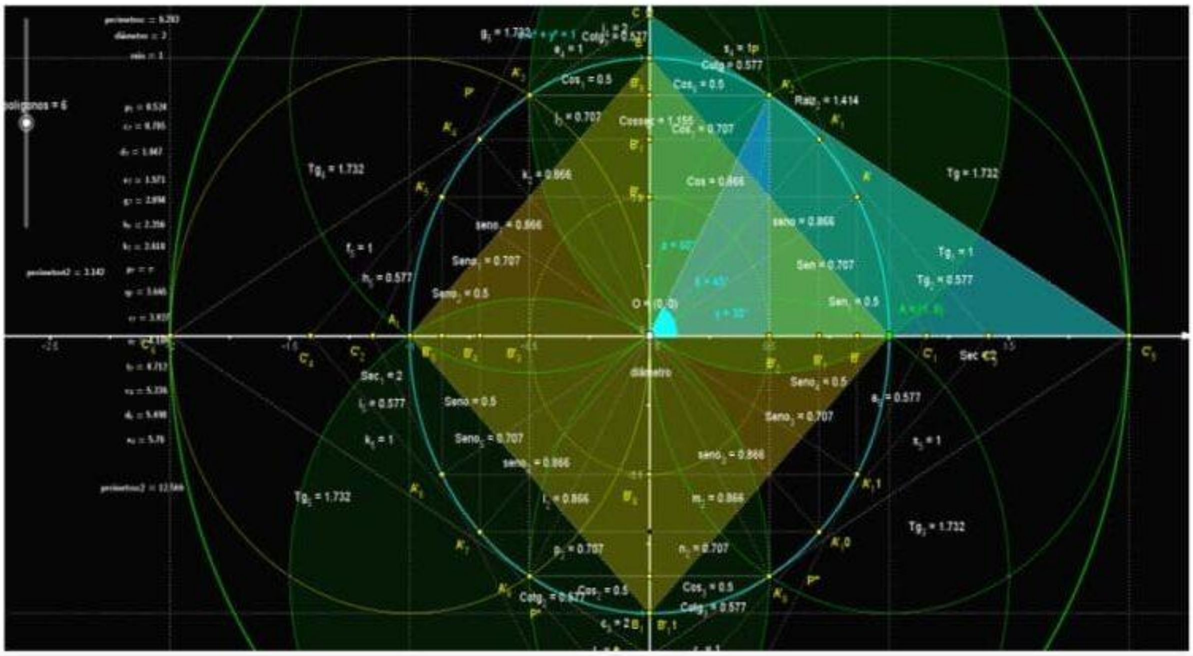
Figure 1. Demonstration of the trigonometric circle in the Cartesian plane with radius=1



Circunferência	Diâmetro	Divisão
6.2832	2	3.1416
Lados polígonos	Lados polígonos * p	Divisão
0.5176	6.2117	1.0115
Arco radianos	Arco radianos * p	Divisão
0.5236	6.2832	1

Grau	Radianos	valores	sen	valores	cos	valores	tg	valores	cotg	valores	sec	valores	cossec	valores
0°	0	0	0	0	1	1	0	0	não	não	1	1	não	não
30°	π/6	0.5236	1/2	0.5	√3/2	0.866	√3/3	0.866	√3	1.7321	2 * √3/3	1.1547	2	2
45°	π/4	0.7854	√2/2	0.7071	√2/2	0.7071	1	1	1	1	√2	1.4142	√2	1.4142
60°	π/3	1.0472	√3/2	0.866	1/2	0.5	√3	1.7321	√3/3	0.866	2	2	2 * √3/3	1.1547
90°	π/2	1.5708	1	1	0	0	não	não	0	0	não	não	1	1
120°	2 * π/3	2.0944	√3/2	0.866	−1/2	−0.5	−√3	−1.7321	−√3/3	−0.866	−2	−2	2 * √3/3	1.1547
135°	3 * π/4	2.3562	√2/2	0.7071	−√2/2	−0.7071	−1	−1	−1	−1	−√2	−1.4142	√2	1.4142
150°	5 * π/6	2.618	1/2	0.5	−√3/2	−0.866	−√3/3	−1.7321	−√3	−1.7321	−2 * √3/3	−1.1547	2	2
180°	π	π	0	0	−1	−1	0	0	não	não	−1	−1	não	não
210°	7 * π/6	3.6652	−1/2	−0.5	−√3/2	−0.866	√3/3	0.866	√3	1.7321	−2 * √3/3	−1.1547	−2	−2
225°	5 * π/4	3.927	−√2/2	−0.7071	−√2/2	−0.7071	1	1	1	1	−√2	−1.4142	−√2	−1.4142
240°	4 * π/3	4.1888	−√3/2	−0.866	−1/2	−0.5	√3	1.7321	√3/3	0.866	−2	−2	−2 * √3/3	−1.1547
270°	3π/2	4.7124	−1	−1	0	0	0	0	0	0	não	não	−1	−1
300°	5 * π/3	5.236	−√3/2	−0.866	1/2	0.5	−√3	−1.7321	−√3	−1.7321	2	2	−2 * √3/3	−1.1547
315°	7 * π/4	5.4978	−√2/2	−0.7071	√2/2	0.7071	−1	−1	−1	−1	√2	1.4142	−√2	−1.4142
330°	11 * π/6	5.7596	−1/2	−0.5	√3/2	0.866	−√3/3	−0.866	−√3/3	−0.866	2 * √3/3	1.1547	−2	−2
360°	2 * π	6.2832	0	0	1	1	0	0	não	não	1	1	não	não

Source: Created by the author using GeoGebra (2021).

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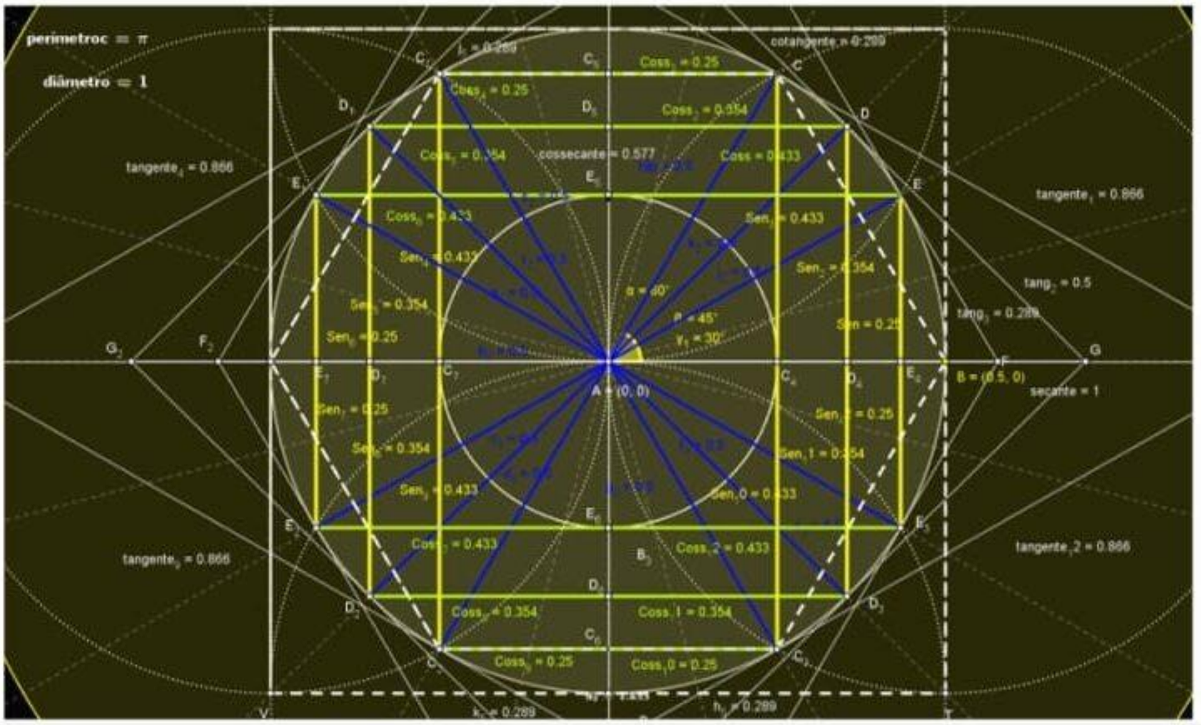
Figure 2. Demonstration of the trigonometric circle in the Cartesian plane with radius=**0.5**



TRIGONOMETRIA DINÂMICA RACIONAL PERIÓDICA INFINITA

TRIGONOMETRIA	RACIONAL	PERIÓDICA	Circunferências	π	Diâmetros	1	π
Ângulos	Seno	Cosseno	Tangente	Cotangente	Secante	Cossecante	Radianos
o°	0	0.5	0	¬	0.5	¬	0
30°	0.25	0.4330127019	0.2886751346	0.8660254038	0.5773502692	1	0.2617993878
45°	0.3535533906	0.3535533906	0.5	0.5	0.7071067812	0.7071067812	0.3926990817
60°	0.4330127019	0.25	0.8660254038	0.2886751346	1	0.5773502692	0.5235987756
90°	0.5	0	¬	0	¬	0.5	0.7853981634
120°	0.4330127019	−0.25	−0.8660254038	−0.2886751346	−1	0.5773502692	1.0471975512
135°	0.3535533906	−0.3535533906	−0.5	−0.5	−0.7071067812	0.7071067812	1.1780972451
150°	0.25	−0.4330127019	−0.2886751346	−0.8660254038	−0.5773502692	1	1.308996939
180°	0	−0.5	0	¬	−0.5	¬	1.5707963268
210°	−0.25	−0.4330127019	0.8660254038	0.8660254038	−0.5773502692	−1	1.8325957146
225°	−0.3535533906	−0.3535533906	0.5	0.5	−0.7071067812	−0.7071067812	1.9634954085
240°	−0.4330127019	−0.25	0.2886751346	0.2886751346	−1	−0.5773502692	2.0943951024
270°	−0.5	0	¬	0	¬	−0.5	2.3561944902
300°	−0.4330127019	0.25	−0.8660254038	−0.2886751346	1	−0.5773502692	2.617993878
315°	−0.3535533906	0.3535533906	−0.5	−0.5	0.7071067812	−0.7071067812	2.7488935719
330°	−0.25	0.4330127019	−0.2886751346	−0.8660254038	0.5773502692	−1	2.8797932658
360°	0	0.5	0	¬	0.5	¬	π

Source: Created by the author using GeoGebra (2021).

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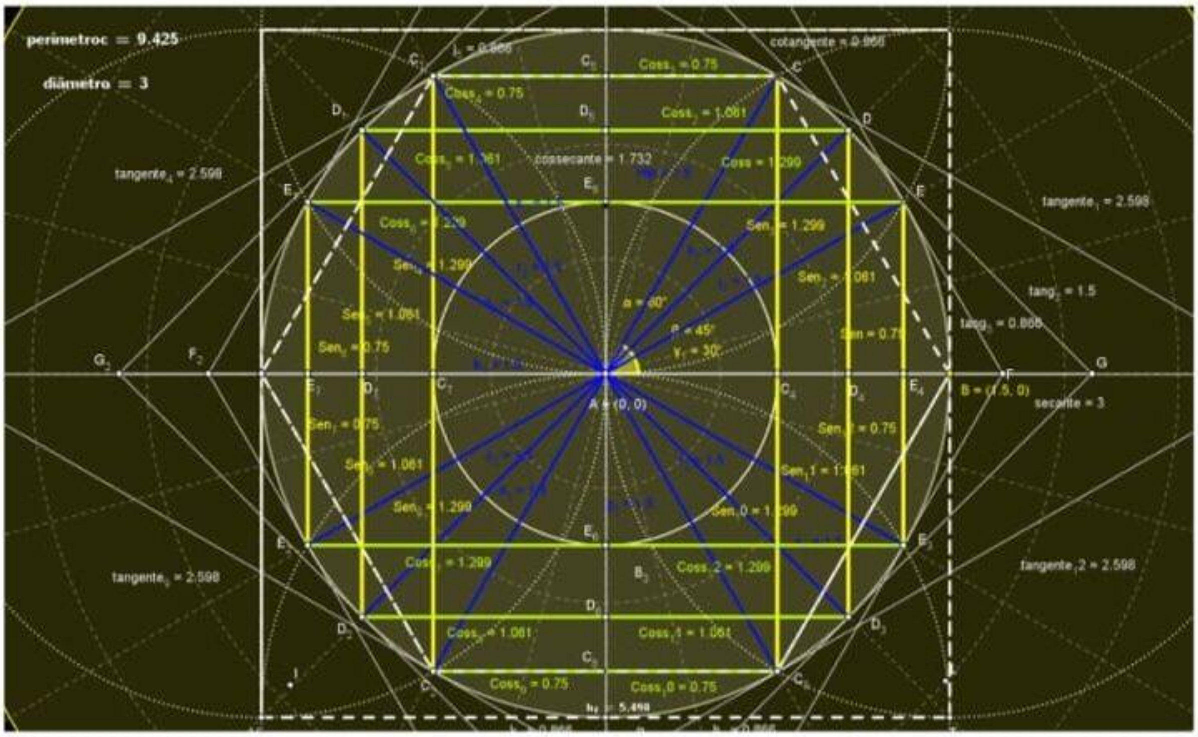
Figure 3. Demonstration of the trigonometric circle in the Cartesian plane with radius=**1.5**



TRIGONOMETRIA DINÂMICA RACIONAL PERIÓDICA INFINITA

TRIGONOMETRIA	RACIONAL	PERIÓDICA	Circunferências	9.4247779608	Diâmetros	3	π
Ângulos	Seno	Cosseno	Tangente	Cotangente	Secante	Cossecante	Radianos
o°	0	1.5	0	¬	1.5	¬	0
30°	0.75	1.2990381057	0.8660254038	2.5980762114	1.7320508076	3	0.7853981634
45°	1.0606601718	1.0606601718	1.5	1.5	2.1213203436	2.1213203436	1.1780972451
60°	1.2990381057	0.75	2.5980762114	0.8660254038	3	1.7320508076	1.5707963268
90°	1.5	0	¬	0	¬	1.5	2.3561944902
120°	1.2990381057	−0.75	−2.5980762114	−0.8660254038	−3	1.7320508076	3.1415926536
135°	1.0606601718	−1.0606601718	−1.5	−1.5	−2.1213203436	2.1213203436	3.5342917353
150°	0.75	−1.2990381057	−0.8660254038	−2.5980762114	−1.7320508076	3	3.926990817
180°	0	−1.5	0	¬	−1.5	¬	4.7123889804
210°	−0.75	−1.2990381057	2.5980762114	2.5980762114	−1.7320508076	−3	5.4977871438
225°	−1.0606601718	−1.0606601718	1.5	1.5	−2.1213203436	−2.1213203436	5.8904862255
240°	−1.2990381057	−0.75	0.8660254038	0.8660254038	−3	−1.7320508076	6.2831853072
270°	−1.5	0	¬	0	¬	−1.5	7.0685834706
300°	−1.2990381057	0.75	−2.5980762114	−0.8660254038	3	−1.7320508076	7.853981634
315°	−1.0606601718	1.0606601718	−1.5	−1.5	2.1213203436	−2.1213203436	8.2466807157
330°	−0.75	1.2990381057	−0.8660254038	−2.5980762114	1.7320508076	−3	8.6393797974
360°	0	1.5	0	¬	1.5	¬	9.4247779608

Source: Created by the author using GeoGebra (2021).

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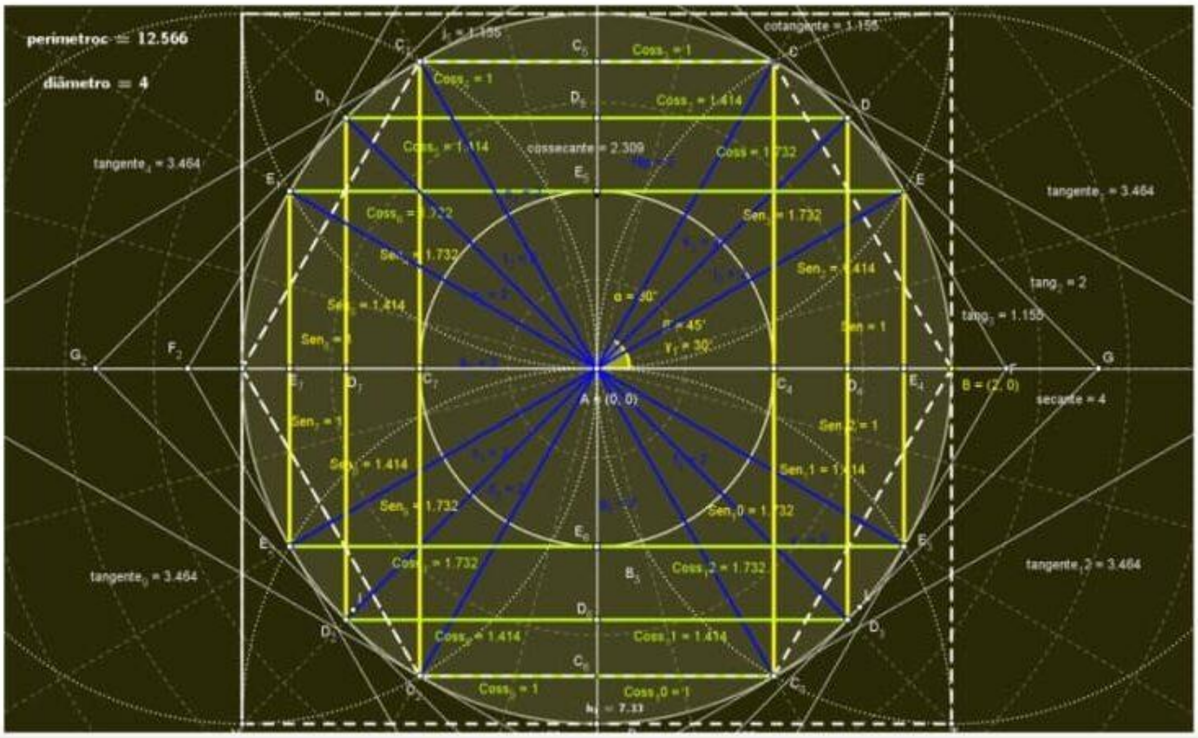
Figure 4. Demonstration of the trigonometric circle in the Cartesian plane with radius=**2**



TRIGONOMETRIA DINÂMICA RACIONAL PERIÓDICA INFINITA

TRIGONOMETRIA	RACIONAL	PERIÓDICA	Circunferências	12.5663706144	Diâmetros	4	π
Ângulos	Seno	Cosseno	Tangente	Cotangente	Secante	Cossecante	Radianos
0°	0	2	0	¬	2	¬	0
30°	1	1.7320508076	1.1547005384	3.4641016151	2.3094010768	4	1.0471975512
45°	1.4142135624	1.4142135624	2	2	2.8284271247	2.8284271247	1.5707963268
60°	1.7320508076	1	3.4641016151	1.1547005384	4	2.3094010768	2.0943951024
90°	2	0	¬	0	¬	2	π
120°	1.7320508076	−1	−3.4641016151	−1.1547005384	−4	2.3094010768	4.1887902048
135°	1.4142135624	−1.4142135624	−2	−2	−2.8284271247	2.8284271247	4.7123889804
150°	1	−1.7320508076	−1.1547005384	−3.4641016151	−2.3094010768	4	5.235987756
180°	0	−2	0	¬	−2	¬	6.2831853072
210°	−1	−1.7320508076	3.4641016151	3.4641016151	−2.3094010768	−4	7.3303828584
225°	−1.4142135624	−1.4142135624	2	2	−2.8284271247	−2.8284271247	7.853981634
240°	−1.7320508076	−1	1.1547005384	1.1547005384	−4	−2.3094010768	8.3775804096
270°	−2	0	¬	0	¬	−2	9.4247779608
300°	−1.7320508076	1	−3.4641016151	−1.1547005384	4	−2.3094010768	10.471975512
315°	−1.4142135624	1.4142135624	−2	−2	2.8284271247	−2.8284271247	10.9955742876
330°	−1	1.7320508076	−1.1547005384	−3.4641016151	2.3094010768	−4	11.5191730632
360°	0	2	0	¬	2	¬	12.5663706144

Source: Created by the author using GeoGebra (2021).

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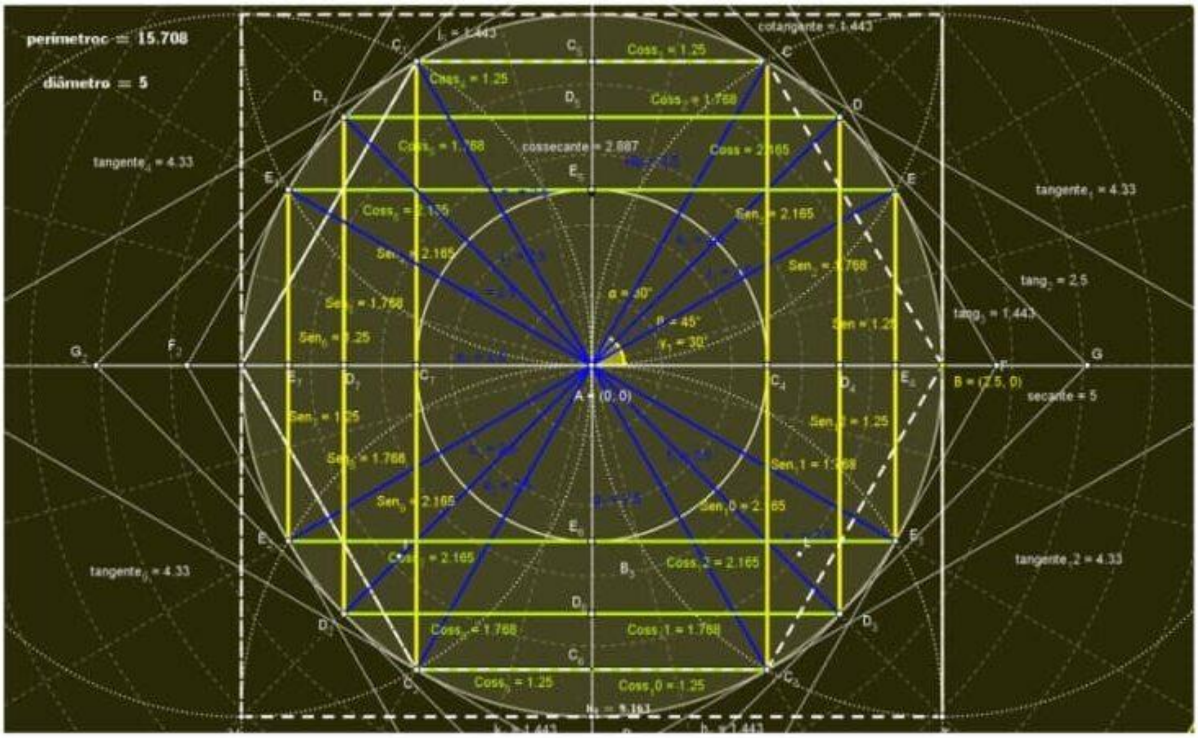
Figure 5. Demonstration of the trigonometric circle in the Cartesian plane with radius=**2.5**



TRIGONOMETRIA DINÂMICA RACIONAL PERIÓDICA INFINITA

TRIGONOMETRIA	RACIONAL	PERIÓDICA	Circunferências	15.7079632679	Diâmetros	5	π
Ângulos	Seno	Cosseno	Tangente	Cotangente	Secante	Cossecante	Radianos
0°	0	2.5	0	¬	2.5	¬	0
30°	1.25	2.1650635095	1.443375673	4.3301270189	2.8867513459	5	1.308996939
45°	1.767766953	1.767766953	2.5	2.5	3.5355339059	3.5355339059	1.9634954085
60°	2.1650635095	1.25	4.3301270189	1.443375673	5	2.8867513459	2.617993878
90°	2.5	0	¬	0	¬	2.5	3.926990817
120°	2.1650635095	−1.25	−4.3301270189	−1.443375673	−5	2.8867513459	5.235987756
135°	1.767766953	−1.767766953	−2.5	−2.5	−3.5355339059	3.5355339059	5.8904862255
150°	1.25	−2.1650635095	−1.443375673	−4.3301270189	−2.8867513459	5	6.544984695
180°	0	−2.5	0	¬	−2.5	¬	7.853981634
210°	−1.25	−2.1650635095	4.3301270189	4.3301270189	−2.8867513459	−5	9.162978573
225°	−1.767766953	−1.767766953	2.5	2.5	−3.5355339059	−3.5355339059	9.8174770425
240°	−2.1650635095	−1.25	1.443375673	1.443375673	−5	−2.8867513459	10.471975512
270°	−2.5	0	¬	0	¬	−2.5	11.780972451
300°	−2.1650635095	1.25	−4.3301270189	−1.443375673	5	−2.8867513459	13.08996939
315°	−1.767766953	1.767766953	−2.5	−2.5	3.5355339059	−3.5355339059	13.7444678595
330°	−1.25	2.1650635095	−1.443375673	−4.3301270189	2.8867513459	−5	14.398966329
360°	0	2.5	0	¬	2.5	¬	15.7079632679

Source: Created by the author using GeoGebra (2021).

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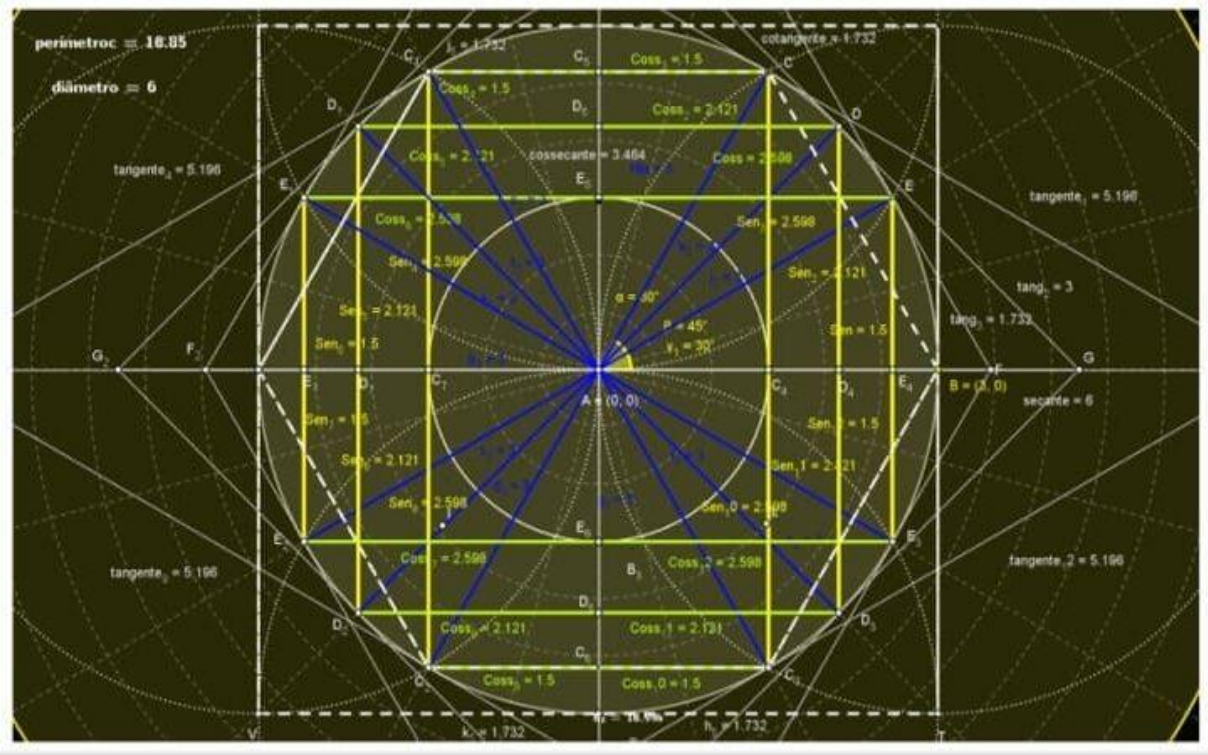
Figure 6. Demonstration of the trigonometric circle in the Cartesian plane with radius=**3**



TRIGONOMETRIA DINÂMICA RACIONAL PERIÓDICA INFINITA

TRIGONOMETRIA	RACIONAL	PERIÓDICA	Circunferências	18.8495559215	Diâmetros	6	π
Ângulos	Seno	Cosseno	Tangente	Cotangente	Secante	Cossecante	Radianos
0°	0	3	0	¬	3	¬	0
30°	1.5	2.5980762114	1.7320508076	5.1961524227	3.4641016151	6	1.5707963268
45°	2.1213203436	2.1213203436	3	3	4.2426406871	4.2426406871	2.3561944902
60°	2.5980762114	1.5	5.1961524227	1.7320508076	6	3.4641016151	π
90°	3	0	¬	0	¬	3	4.7123889804
120°	2.5980762114	−1.5	−5.1961524227	−1.7320508076	−6	3.4641016151	6.2831853072
135°	2.1213203436	−2.1213203436	−3	−3	−4.2426406871	4.2426406871	7.0685834706
150°	1.5	−2.5980762114	−1.7320508076	−5.1961524227	−3.4641016151	6	7.853981634
180°	0	−3	0	¬	−3	¬	9.4247779608
210°	−1.5	−2.5980762114	5.1961524227	5.1961524227	−3.4641016151	−6	10.9955742876
225°	−2.1213203436	−2.1213203436	3	3	−4.2426406871	−4.2426406871	11.780972451
240°	−2.5980762114	−1.5	1.7320508076	1.7320508076	−6	−3.4641016151	12.5663706144
270°	−3	0	¬	0	¬	−3	14.1371669412
300°	−2.5980762114	1.5	−5.1961524227	−1.7320508076	6	−3.4641016151	15.7079632679
315°	−2.1213203436	2.1213203436	−3	−3	4.2426406871	−4.2426406871	16.4933614313
330°	−1.5	2.5980762114	−1.7320508076	−5.1961524227	3.4641016151	−6	17.2787595947
360°	0	3	0	¬	3	¬	18.8495559215

Source: Created by the author using GeoGebra (2021).

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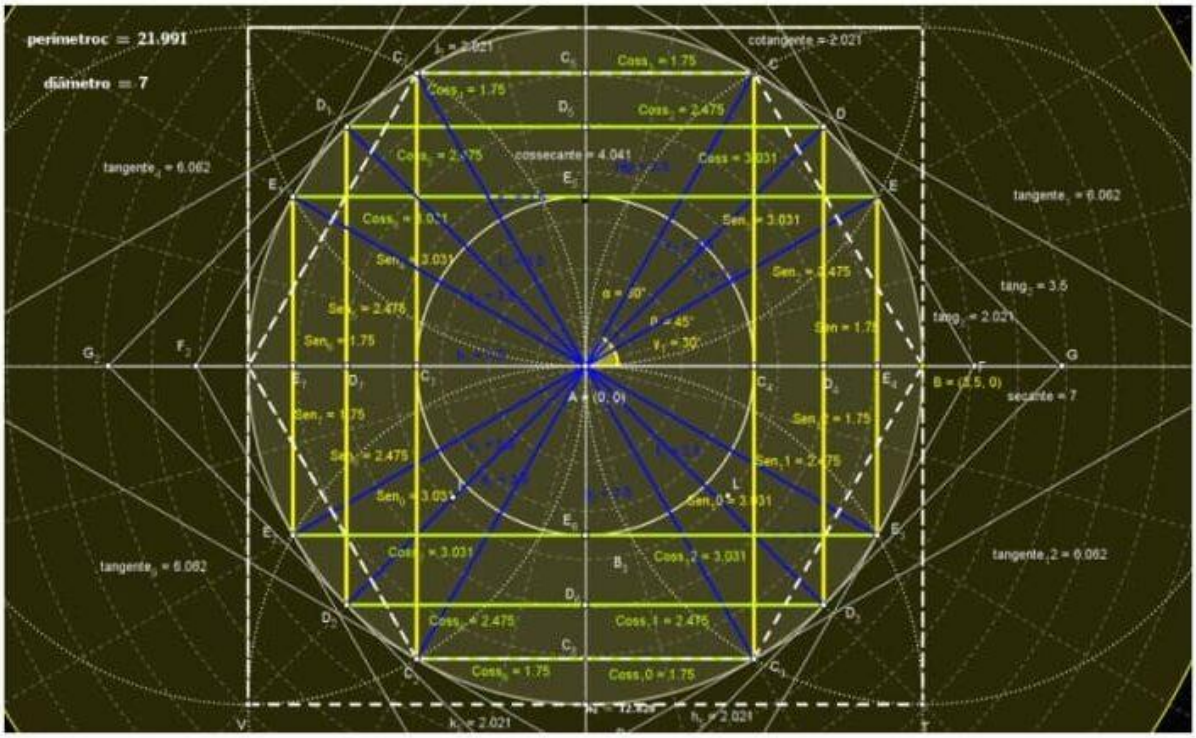
Figure 7. Demonstration of the trigonometric circle in the Cartesian plane with radius=**3.5**



TRIGONOMETRIA DINÂMICA RACIONAL PERIÓDICA INFINITA

TRIGONOMETRIA	RACIONAL	PERIÓDICA	Circunferências	21.9911485751	Diâmetros	7	π
Ângulos	Seno	Cosseno	Tangente	Cotangente	Secante	Cossecante	Radianos
0°	0	3.5	0	¬	3.5	¬	0
30°	1.75	3.0310889132	2.0207259422	6.0621778265	4.0414518843	7	1.8325957146
45°	2.4748737342	2.4748737342	3.5	3.5	4.9497474683	4.9497474683	2.7488935719
60°	3.0310889132	1.75	6.0621778265	2.0207259422	7	4.0414518843	3.6651914292
90°	3.5	0	¬	0	¬	3.5	5.4977871438
120°	3.0310889132	−1.75	−6.0621778265	−2.0207259422	−7	4.0414518843	7.3303828584
135°	2.4748737342	−2.4748737342	−3.5	−3.5	−4.9497474683	4.9497474683	8.2466807157
150°	1.75	−3.0310889132	−2.0207259422	−6.0621778265	−4.0414518843	7	9.162978573
180°	0	−3.5	0	¬	−3.5	¬	10.9955742876
210°	−1.75	−3.0310889132	6.0621778265	6.0621778265	−4.0414518843	−7	12.8281700022
225°	−2.4748737342	−2.4748737342	3.5	3.5	−4.9497474683	−4.9497474683	13.7444678595
240°	−3.0310889132	−1.75	2.0207259422	2.0207259422	−7	−4.0414518843	14.6607657168
270°	−3.5	0	¬	0	¬	−3.5	16.4933614313
300°	−3.0310889132	1.75	−6.0621778265	−2.0207259422	7	−4.0414518843	18.3259571459
315°	−2.4748737342	2.4748737342	−3.5	−3.5	4.9497474683	−4.9497474683	19.2422550032
330°	−1.75	3.0310889132	−2.0207259422	−6.0621778265	4.0414518843	−7	20.1585528605
360°	0	3.5	0	¬	3.5	¬	21.9911485751

Source: Created by the author using GeoGebra (2021).

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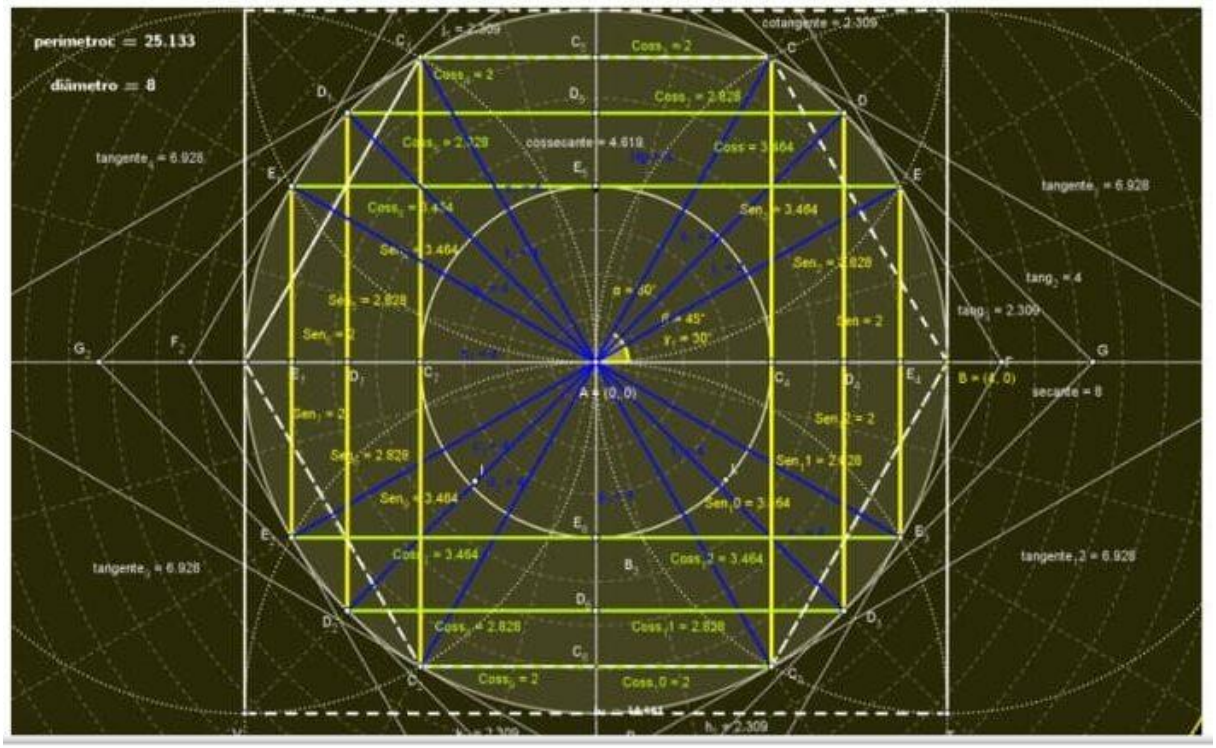
Figure 8. Demonstration of the trigonometric circle in the Cartesian plane with radius=**4**

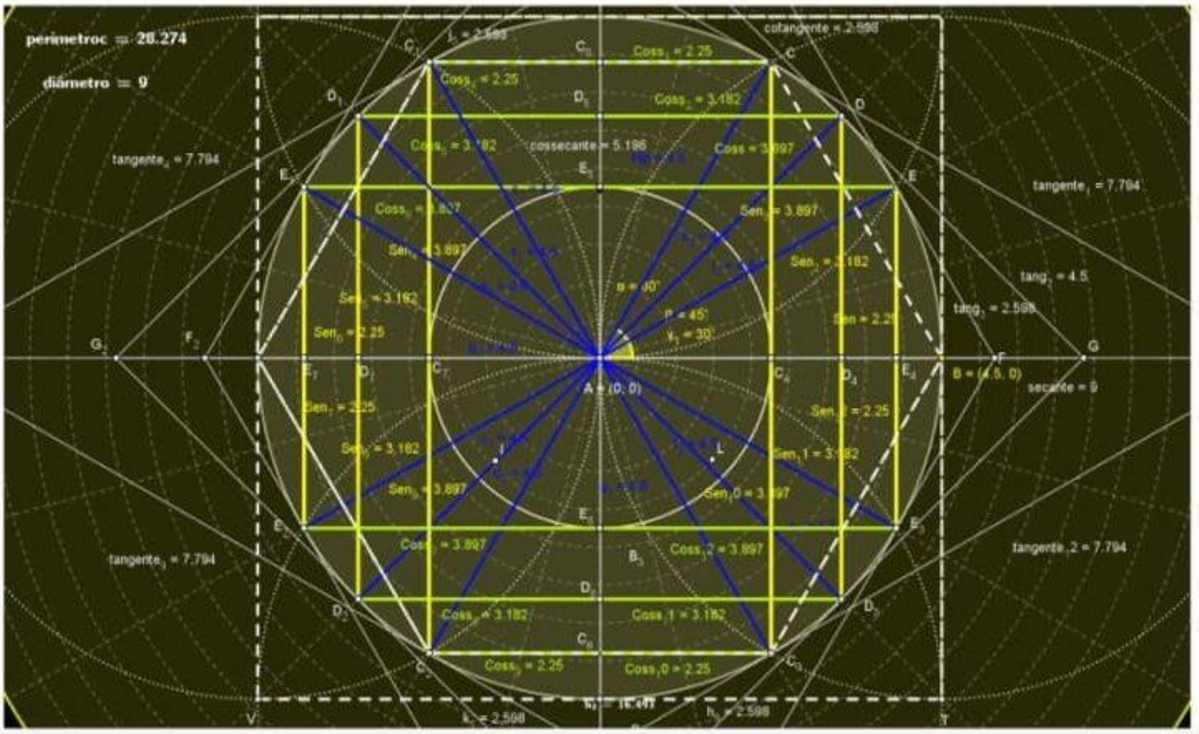


TRIGONOMETRIA DINÂMICA RACIONAL PERIÓDICA INFINITA

TRIGONOMETRIA	RACIONAL	PERIÓDICA	Circunferências	25.1327412287	Diâmetros	8	π
Ângulos	Seno	Cosseno	Tangente	Cotangente	Secante	Cossecante	Radianos
o°	0	4	0	¬	4	¬	0
30°	2	3.4641016151	2.3094010768	6.9282032303	4.6188021535	8	2.0943951024
45°	2.8284271247	2.8284271247	4	4	5.6568542495	5.6568542495	π
60°	3.4641016151	2	6.9282032303	2.3094010768	8	4.6188021535	4.1887902048
90°	4	0	¬	0	¬	4	6.2831853072
120°	3.4641016151	−2	−6.9282032303	−2.3094010768	−8	4.6188021535	8.3775804096
135°	2.8284271247	−2.8284271247	−4	−4	−5.6568542495	5.6568542495	9.4247779608
150°	2	−3.4641016151	−2.3094010768	−6.9282032303	−4.6188021535	8	10.471975512
180°	0	−4	0	¬	−4	¬	12.5663706144
210°	−2	−3.4641016151	6.9282032303	6.9282032303	−4.6188021535	−8	14.6607657168
225°	−2.8284271247	−2.8284271247	4	4	−5.6568542495	−5.6568542495	15.7079632679
240°	−3.4641016151	−2	2.3094010768	2.3094010768	−8	−4.6188021535	16.7551608191
270°	−4	0	¬	0	¬	−4	18.8495559215
300°	−3.4641016151	2	−6.9282032303	−2.3094010768	8	−4.6188021535	20.9439510239
315°	−2.8284271247	2.8284271247	−4	−4	5.6568542495	−5.6568542495	21.9911485751
330°	−2	3.4641016151	−2.3094010768	−6.9282032303	4.6188021535	−8	23.0383461263
360°	0	4	0	¬	4	¬	25.1327412287

Source: Created by the author using GeoGebra (2021).

Figure 9. Demonstration of the trigonometric circle in the Cartesian plane with radius=**4.5**



TRIGONOMETRIA DINÂMICA RACIONAL PERIÓDICA INFINITA

TRIGONOMETRIA	RACIONAL	PERIÓDICA	Circunferências	28.2743338823	Diâmetros	9	π
Ângulos	Seno	Cosseno	Tangente	Cotangente	Secante	Cossecante	Radianos
0°	0	4.5	0	¬	4.5	¬	0
30°	2.25	3.897114317	2.5980762114	7.7942286341	5.1961524227	9	2.3561944902
45°	3.1819805153	3.1819805153	4.5	4.5	6.3639610307	6.3639610307	3.5342917353
60°	3.897114317	2.25	7.7942286341	2.5980762114	9	5.1961524227	4.7123889804
90°	4.5	0	¬	0	¬	4.5	7.0685834706
120°	3.897114317	−2.25	−7.7942286341	−2.5980762114	−9	5.1961524227	9.4247779608
135°	3.1819805153	−3.1819805153	−4.5	−4.5	−6.3639610307	6.3639610307	10.6028752059
150°	2.25	−3.897114317	−2.5980762114	−7.7942286341	−5.1961524227	9	11.780972451
180°	0	−4.5	0	¬	−4.5	¬	14.1371669412
210°	−2.25	−3.897114317	7.7942286341	7.7942286341	−5.1961524227	−9	16.4933614313
225°	−3.1819805153	−3.1819805153	4.5	4.5	−6.3639610307	−6.3639610307	17.6714586764
240°	−3.897114317	−2.25	2.5980762114	2.5980762114	−9	−5.1961524227	18.8495559215
270°	−4.5	0	¬	0	¬	−4.5	21.2057504117
300°	−3.897114317	2.25	−7.7942286341	−2.5980762114	9	−5.1961524227	23.5619449019
315°	−3.1819805153	3.1819805153	−4.5	−4.5	6.3639610307	−6.3639610307	24.740042147
330°	−2.25	3.897114317	−2.5980762114	−7.7942286341	5.1961524227	−9	25.9181393921
360°	0	4.5	0	¬	4.5	¬	28.2743338823

Source: Created by the author using GeoGebra (2021).

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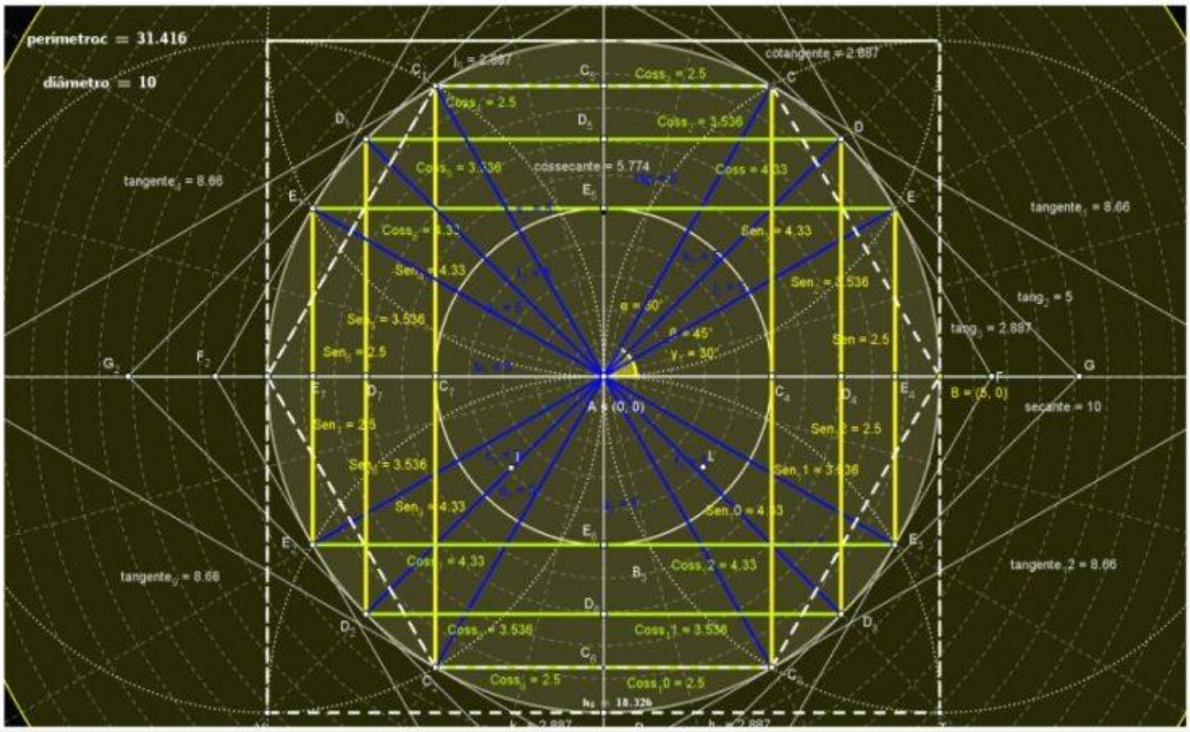
Figure 10. Demonstration of the trigonometric circle in the Cartesian plane with radius=**5**



TRIGONOMETRIA DINÂMICA RACIONAL PERIÓDICA INFINITA

TRIGONOMETRIA	RACIONAL	PERIÓDICA	Circunferências	31.4159265359	Diâmetros	10	π
Ângulos	Seno	Cosseno	Tangente	Cotangente	Secante	Cossecante	Radianos
o°	0	5	0	¬	5	¬	0
30°	2.5	4.3301270189	2.8867513459	8.6602540378	5.7735026919	10	2.617993878
45°	3.5355339059	3.5355339059	5	5	7.0710678119	7.0710678119	3.926990817
60°	4.3301270189	2.5	8.6602540378	2.8867513459	10	5.7735026919	5.235987756
90°	5	0	¬	0	¬	5	7.853981634
120°	4.3301270189	−2.5	−8.6602540378	−2.8867513459	−10	5.7735026919	10.471975512
135°	3.5355339059	−3.5355339059	−5	−5	−7.0710678119	7.0710678119	11.780972451
150°	2.5	−4.3301270189	−2.8867513459	−8.6602540378	−5.7735026919	10	13.08996939
180°	0	−5	0	¬	−5	¬	15.7079632679
210°	−2.5	−4.3301270189	8.6602540378	8.6602540378	−5.7735026919	−10	18.3259571459
225°	−3.5355339059	−3.5355339059	5	5	−7.0710678119	−7.0710678119	19.6349540849
240°	−4.3301270189	−2.5	2.8867513459	2.8867513459	−10	−5.7735026919	20.9439510239
270°	−5	0	¬	0	¬	−5	23.5619449019
300°	−4.3301270189	2.5	−8.6602540378	−2.8867513459	10	−5.7735026919	26.1799387799
315°	−3.5355339059	3.5355339059	−5	−5	7.0710678119	−7.0710678119	27.4889357189
330°	−2.5	4.3301270189	−2.8867513459	−8.6602540378	5.7735026919	−10	28.7979326579
360°	0	5	0	¬	5	¬	31.4159265359

Source: Created by the author using GeoGebra (2021).

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2.2 TRIGONOMETRIC CIRCUMFERENCE IN THE ISOMETRIC PLANE

The circumference is a set of points in a plane where the distance to a specific point in that plane is equal to a given distance. The given point is the center, and the given distance is the radius (r) of the circumference. Thus, given a plane α, a point O (0,0), at a distance Radius (r), we have: C: $x^2 + y^2 = IR$ (real numbers).

Figure 11. Trigonometric circle in the isometric plane diameter= $(\sqrt{3}=1.732\ 050...)$

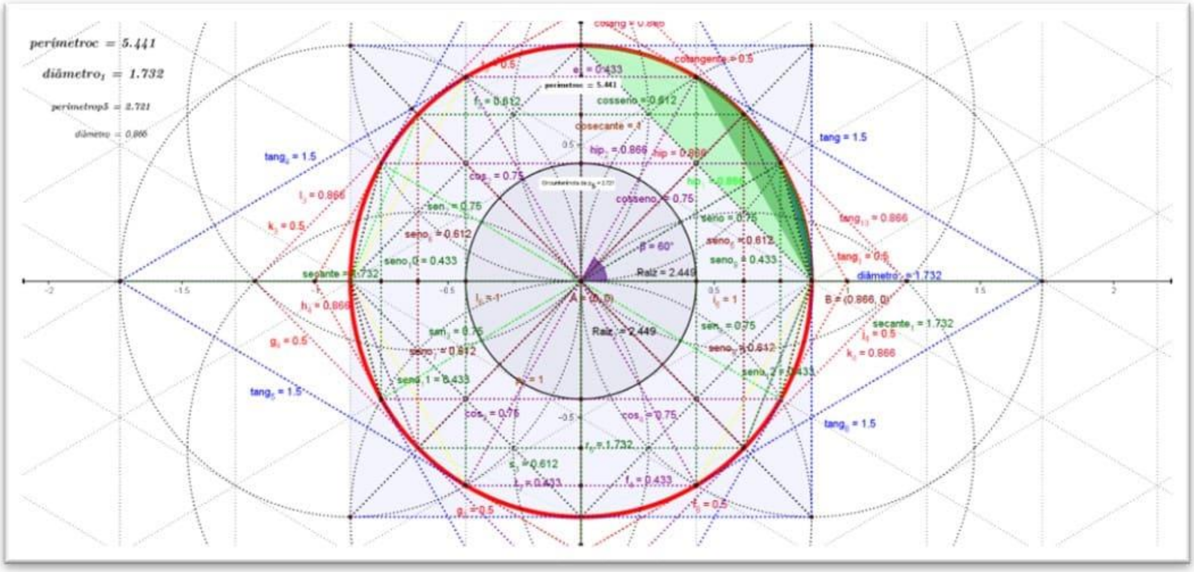

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Table 1. Demonstration in the isometric plane of infinite rational periodic fig. 11

Circunferências	Diâmetros	Divisões racionais
5.441 398 092 702 66...	1.732 050 807 568...	π=3.141 592 653 589 389...
Raio	0.866 025 403 784.	IM(f)= {y.∈ R ↑ {-$\sqrt{3/2}$, $\sqrt{3/2}$}

TRIGONOMETRIA DINÂMICA RACIONAL PERIÓDICA INFINITA

TRIGONOMETRIA	RACIONAL	PERIÓDICA	Circunferências	5.4413980927	Diâmetros	1.7320508076	π
Ângulos	Seno	Cosseno	Tangente	Cotangente	Secante	Cossecante	Radianos
o°	0	0.8660254038	0	¬	0.8660254038	¬	0
30°	0.4330127019	0.75	0.5	1.5	1	1.7320508076	0.4534498411
45°	0.6123724357	0.6123724357	0.8660254038	0.8660254038	1.2247448714	1.2247448714	0.6801747616
60°	0.75	0.4330127019	1.5	0.5	1.7320508076	1	0.9068996821
90°	0.8660254038	0	¬	0	¬	0.8660254038	1.3603495232
120°	0.75	−0.4330127019	−1.5	−0.5	−1.7320508076	1	1.8137993642
135°	0.6123724357	−0.6123724357	−0.8660254038	−0.8660254038	−1.2247448714	1.2247448714	2.0405242848
150°	0.4330127019	−0.75	−0.5	−1.5	−1	1.7320508076	2.2672492053
180°	0	−0.8660254038	0	¬	−0.8660254038	¬	2.7206990464
210°	−0.4330127019	−0.75	1.5	1.5	−1	−1.7320508076	3.1741488874
225°	−0.6123724357	−0.6123724357	0.8660254038	0.8660254038	−1.2247448714	−1.2247448714	3.4008738079
240°	−0.75	−0.4330127019	0.5	0.5	−1.7320508076	−1	3.6275987285
270°	−0.8660254038	0	¬	0	¬	−0.8660254038	4.0810485695
300°	−0.75	0.4330127019	−1.5	−0.5	1.7320508076	−1	4.5344984106
315°	−0.6123724357	0.6123724357	−0.8660254038	−0.8660254038	1.2247448714	−1.2247448714	4.7612233311
330°	−0.4330127019	0.75	−0.5	−1.5	1	−1.7320508076	4.9879482516
360°	0	0.8660254038	0	¬	0.8660254038	¬	5.4413980927

Source: Created by the author using GeoGebra (2021).[/caption]

2.3 SPECIAL VALUES

From convenient right triangles, the definitions of sine, cosine, tangent, cotangent, cosecant, and (secant) allow for the following table of infinite notable periodic rational values (IEZZI, 1993).

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Seno B = $\frac{b}{a}$ = cosseno C = $\frac{b}{a}$

cosseno B = $\frac{c}{a}$ = Seno C = $\frac{c}{a}$

Tangente B = $\frac{b}{c}$ = cotangente C = $\frac{b}{c}$

cotangente B = $\frac{c}{b}$ =Tangente C = $\frac{c}{b}$

Secante B = $\frac{a}{c}$ =cossecante C = $\frac{a}{c}$

cossecante B = $\frac{a}{b}$ = Secante C = $\frac{a}{b}$

(x)	sen.(x)	valores	cos.(x)	valores	Tang(x)	valores
30º	$\frac{1}{2}$	=0.500 000 000	$\frac{\sqrt{3}}{2}$	=0.866 025 40	$\frac{\sqrt{3}}{3}$	=0.577 350
45º	$\frac{\sqrt{2}}{2}$	=0.707 106 781	$\frac{\sqrt{2}}{2}$	=0.707 106 78	1	=1.000 000
60º	$\frac{\sqrt{3}}{2}$	=0.866 025 403	$\frac{1}{2}$	=0.500 000 00	$\sqrt{3}$	=1.732 050
(x)	cot. (x)	valores	cossec(x)	valores	Sec.(x)	valores
30º	$\sqrt{3}$	=1.732 050 808	= 2.	=2.000 000 00	$\frac{2\sqrt{3}}{3}$	=1.154 700
45º	1	=1.000 000 000	$\sqrt{2}$	=1.414 213 56	$\sqrt{2}$	=1.414 213
60º	$\frac{\sqrt{3}}{3}$	=0.577 350 269	$\frac{2\sqrt{3}}{3}$	=1.154 700 53	= 2.	=2.000 000

Source: Created by the author using GeoGebra (2021).

2.4 TRIGONOMETRIC FUNCTIONS

The trigonometric functions on the trigonometric circle are associated with the four axes, where the trigonometric functions sine, cosine, tangent, cotangent, secant, and cosecant will be defined in the Cartesian plane.

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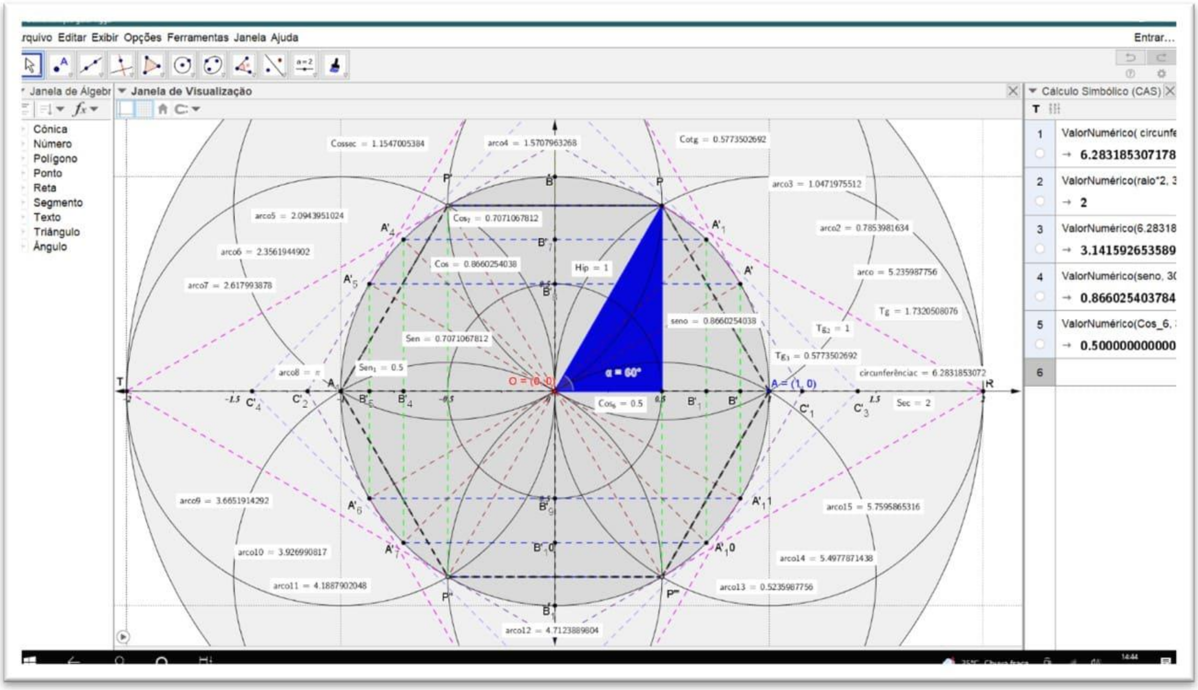
Figure 12. The four axes of the complete trigonometric functions. C: $x^2 + y^2 = 1$



constante	C: x2 + y2 = 1	{3.14159265358938917154368732170691}
Perímetros	{per. c:0,33} =	{6.28318530717877834308737464341382}
Diâmetros	{diâmetro c:0,33} =	{2.00000000000000000000000000000000}
Divisões (racionais)	{divisão. c:0,33} =	{3.14159265358938917154368732170691}
Seno a (60°)	{Seno.} = $\sqrt{3}/2$	{0.86602540378489124679707960950490}
Cosseno (60°)	{cosseno.} =$1/2$	{0.50000000000000000000000000000000}
Tangente (60°)	{tangente =$\sqrt{3}$ }	{1.73205080756978249359415921900981}
Cotangente (60°)	{cotg.} =$\sqrt{3}/3$	{0.57735026919075184872247864978667}
Cossecante (60°)	{cossec. }=$2\sqrt{3}/3$	{1.15470053837864806230715595805022}
Secante (60°)	{secante =$\sqrt{4}$=	{2.00000000000000000000000000000000}
Radianos (60°)	{Arcos} =$\pi/3$	{1.04719755119713465949418080473976}
Radianos (120°)	{Arc.} =$2\pi/3$	{2.09439510239426931898836160947952}
Radianos (240°)	{Arc.} =$4\pi/3$	{4.18879020478585222872491642894254}
Radianos (300°)	{Arc.} = $5\pi/3$	{5.23598775598231528590614553617818}

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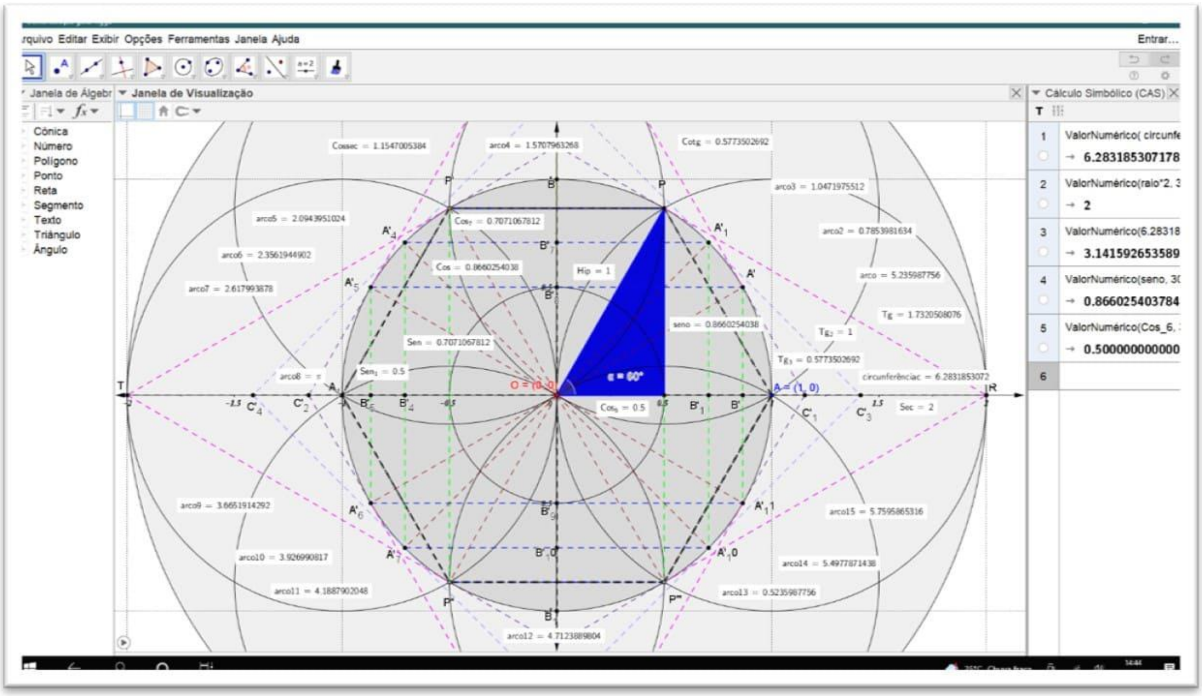
2.4.1 SINE

The definition of sine α as the ratio and proportion between the opposite cathetus to α and the hypotenuse of the right triangle. Therefore, one can define a function from R to R, such that for each x, it associates y=sine α.

From the definition of the function y=f(x)=sine x, it follows that the Domain: D(F)=R; image: IM(f)= {y ∈ R ↑ -1 ≤y≤ 1}

Figure 13. Complete trigonometric functions sine α 60º C: $x^2 + y^2 = 1$



Seno (α=60º)	$\frac{\text{Cateto oposto}}{\text{hipotenusa}}$	$\frac{\sqrt{3}}{2}$ = {0.866025403784891246797079}

Hipotenusa	Seno a (60º)	Cosseno (60º)
A^2 =	B^2 +	C^2
{1.0000000000000000}2	{0.86602540378489124}2	{0.5000000000000000}2
{1.0000000000000000}	{0.75000000000000000}	{0.2500000000000000}

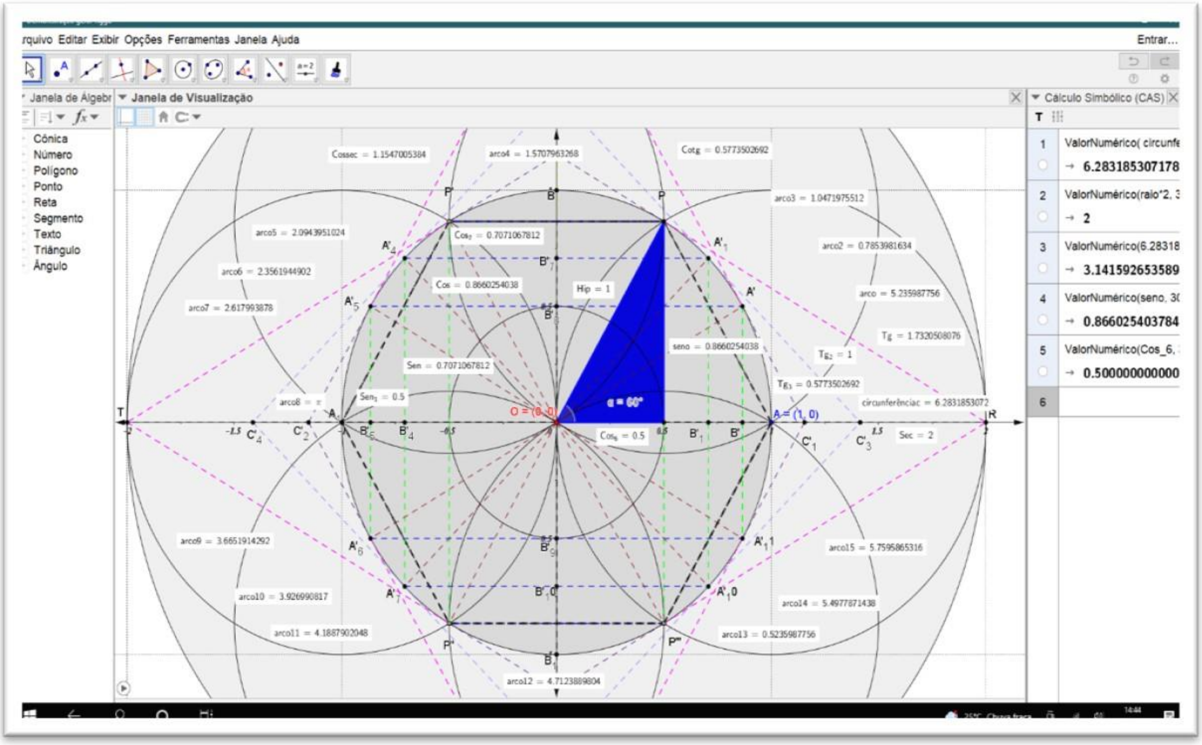
Source: Created by the author using GeoGebra (2021).

2.4.2 COSINE

The definition of cosine α as the ratio and proportion between the adjacent cathetus to α and the hypotenuse of the right triangle. Therefore, one can define a function from R to R, such that for each x, it associates y=cos α.

From the definition of the function y=f(x)=cos x, it follows that the Domain: D(F)=R; image: IM (f x) = {x ∈ R ↑ -1 ≤x≤ 1\}

Figure 14. Demonstration of the four axes of the complete trigonometric functions. Cosine (α=60º) C: $x^2 + y^2 = 1$



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Cosseno (α=60º)	$\frac{\text{Cateto adjacente}}{\text{hipotenusa}}$	1/2= {0,500000000000000000}

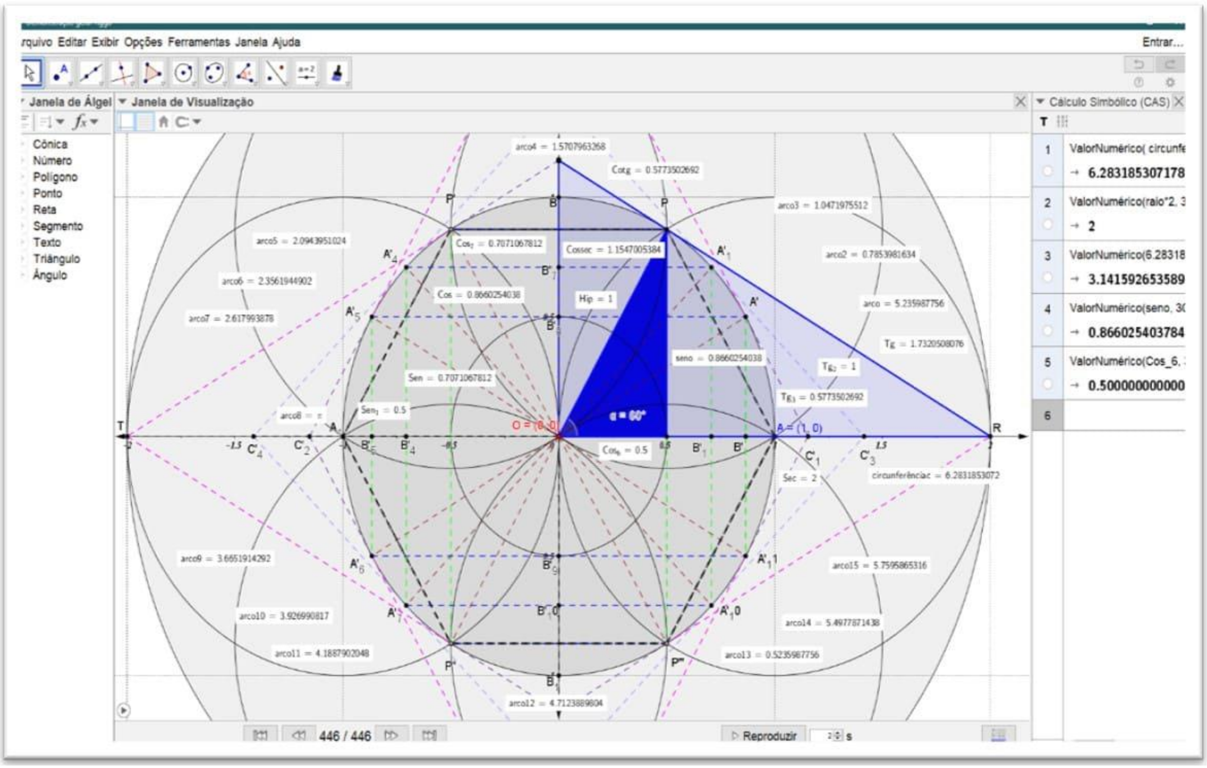
Hipotenusa	Seno a (60º)	Cosseno (60º)
A^2 =	B^2 +	C^2
{1.0000000000000000}2	{0.86602540378489124}2	{0.5000000000000000}2
{1.0000000000000000}	{0.75000000000000000}	{0.2500000000000000}

Source: Created by the author using GeoGebra (2021).

2.4.3 TANGENT

The definition of tangent α as the ratio and proportion between the opposite cathetus (α) and the adjacent cathetus (α), and the hypotenuse of the right triangle.

Figure 15. Demonstration of the four axes of the complete trigonometric functions. Tangent (α=60º) C: $x^2 + y^2 = 1$



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Tangente (α=60º)	$\frac{\textbf{Cateto oposto}}{\textbf{Cateto adjacente}}$	=	$\sqrt{3}$ =**1.732050807569782**

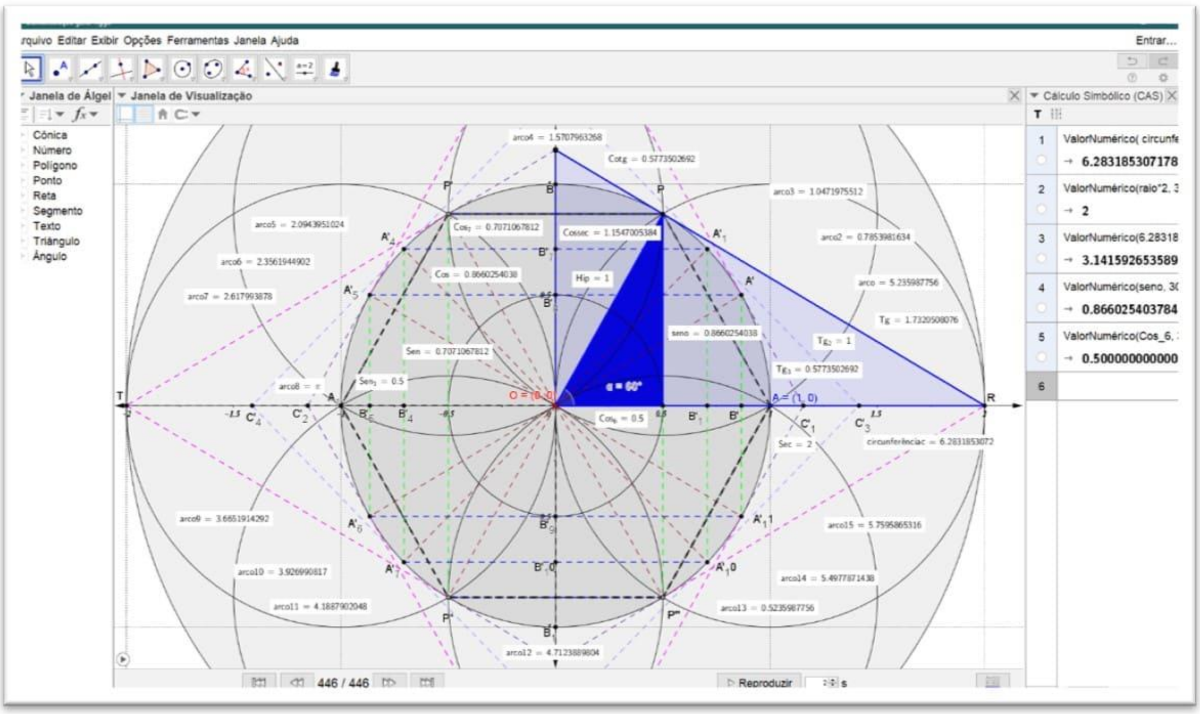
Hipotenusa	**Seno α (60º)**	**Cosseno (60º)**
A^2 =	**B^2 +**	**C^2**
{1.732050807569782}2	**{0.86602540378489124}2**	**{1.5000000000000000}2**
{3.0000000000000000}	**{0.75000000000000000}**	**{2.2500000000000000}**

Source: Created by the author using GeoGebra (2021).

2.4.4 COTANGENT

The definition of cotangent (α) is the inverse of the tangent, being the ratio and proportion between the adjacent cathetus (α) and the opposite cathetus (α), and the hypotenuse of the right triangle.

Figure 16. Demonstration of the four axes of the complete trigonometric functions. Cotangent (α=60º) C: C: x ^{2} + y^2 = 1



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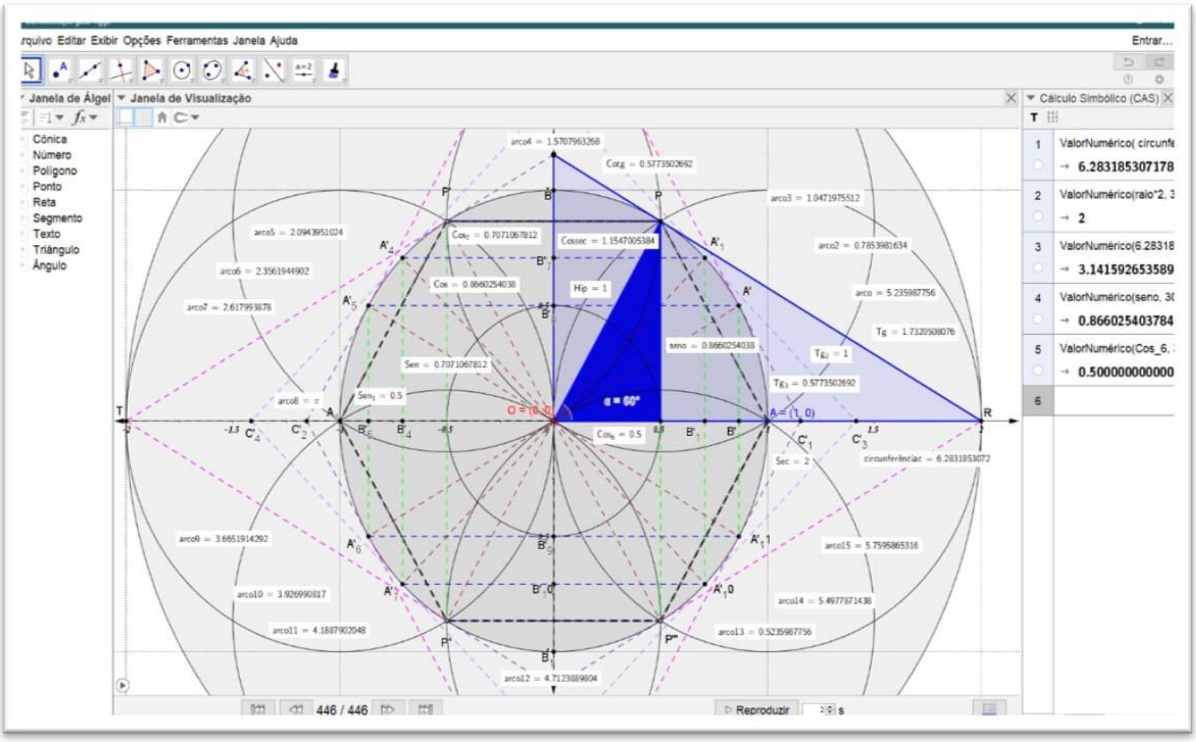
Cotangente (α=60º)	$\frac{\text{Cateto adjacent}}{\text{Cateto oposto}}$	$\frac{\sqrt{3}}{3}$	**=0,5773502691907518487...**

Source: Created by the author using GeoGebra (2021).

2.4.5 COSECANT

The definition of cosecant (α) is the inverse of sine, being the ratio and proportion between (α), the hypotenuse, and the opposite cathetus (α).

Figure 17. Demonstration of the four axes of the trigonometric functions. Cosecant: C: x 2 + y^2 = 1



Cossecante (α=60º)	$\frac{\text{hipotenusa}}{\text{Cateto oposto}}$	$2\sqrt{3}/3$	**1.154700538378648...**

Source: Created by the author using GeoGebra (2021).

2.4.6 SECANT

The definition of secant (α), as the reciprocal of cosine, corresponds to the ratio and proportion of the hypotenuse by the adjacent cathetus.

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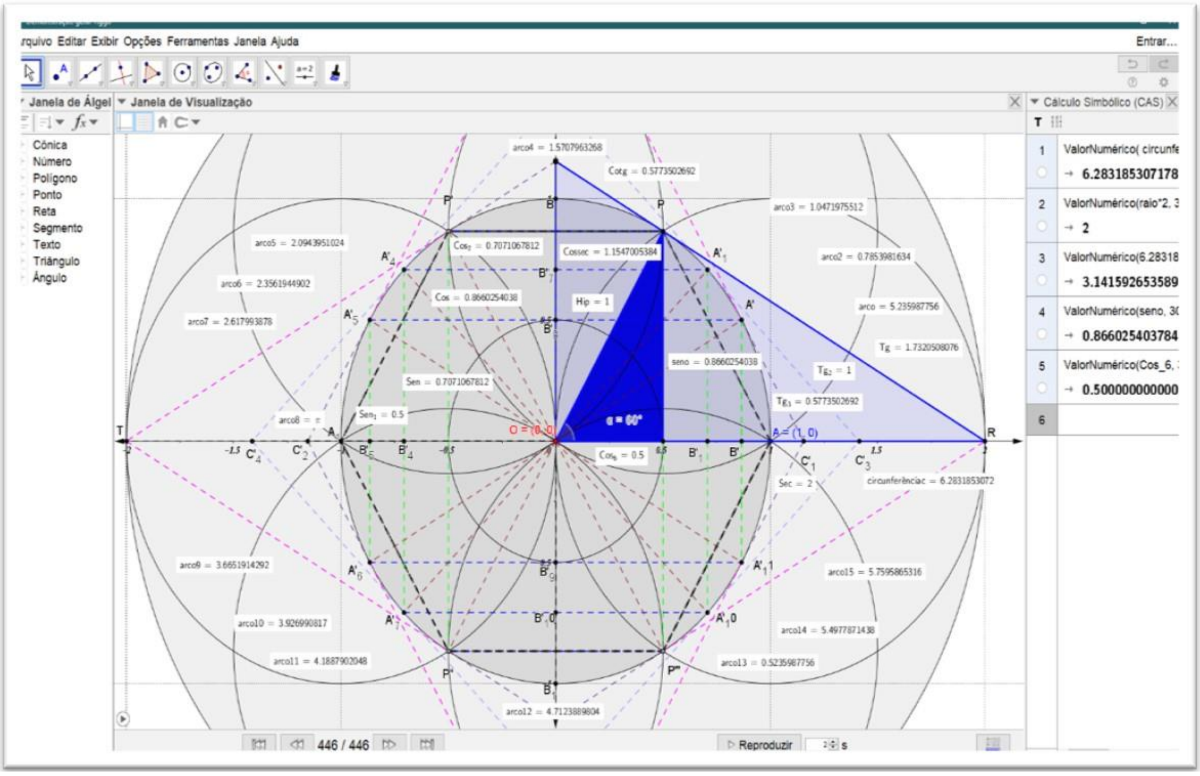
Figure 18. Demonstration of the four axes of the trigonometric functions. Secant: C: x 2 + y^2 = 1



Secante (α=60º)	hipotenusa / Cateto adjacente	$\sqrt{4}$=2.00000000000000000000

Hipotenusa	Seno a (60º)	Cosseno (60º)
A^2 =	B^2 +	C^2
{2.3094010767585}2	{1.1547005383786\48}2	{2}2
{5.333333333333}	{1.33333333333}	{4.}

Source: Created by the author using GeoGebra (2021).

2.5 IRRATIONAL NUMBER Π

Its representation, by the Greek letter π (pi), comes from the Greek word for perimeter (πρειμετρος), and was introduced in 1706 by William Jones (1676-1749), and popularized by Leonhard Euler (1707-1783). Mathematicians from various eras have tried to find a rationality for the constant π. The proof that the constant π is an irrational

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number was made by Johann Lambert in 1761 and Legendre in 1794. The number π is a transcendental number, which was proved by Ferdinand Lindemann in 1882.

2.5.1 CIRCUMFERENCE PERIMETER

The circumference's length, that is, its perimeter c, can be calculated using the equation. C=π*d 2*π*r.

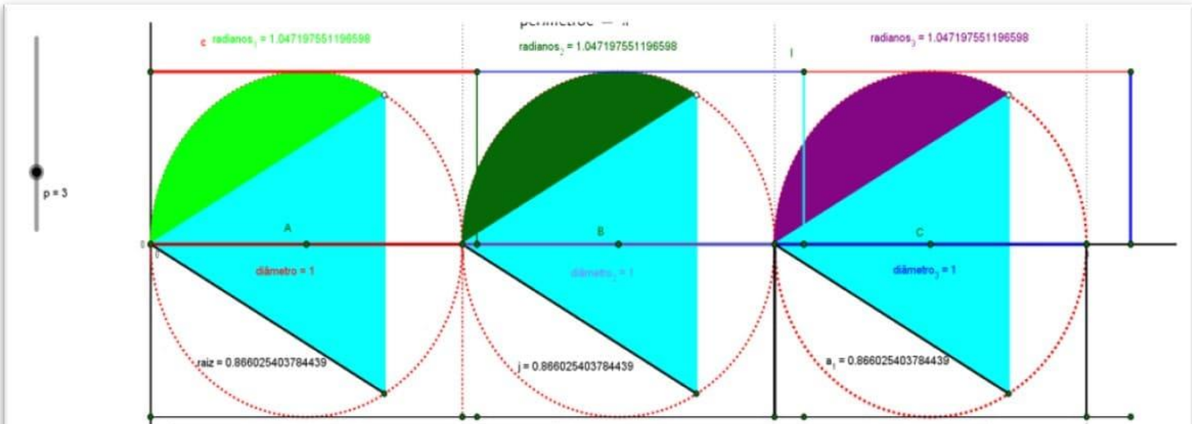
π(pi) constant=**3.141 592 653 589 389 171 543 687 321 706 908 213…**

2.5.2 RATIONALITY OF THE CONSTANT Π

The circle is the internal area (α), delimited on the circumference, which can be calculated using the equation. Área(a)=π*r^2.

Area (2*π) =**6.283 185 307 178 778 343 087 374 643 413 816 427 962…**

Figure 19. Demonstration and comparisons of the trisection of the circumferences' perimeters



Circunferência	π	π	π
diâmetro	1	2	3
Arcos	1.047197551196598	2.094395102393196	3.141592653589794
Diferença	0.047197551196598	0.094395102393196	0.141592653589794
Polígonos	0.866025403784439	1.732050807568878	2.598076211353316

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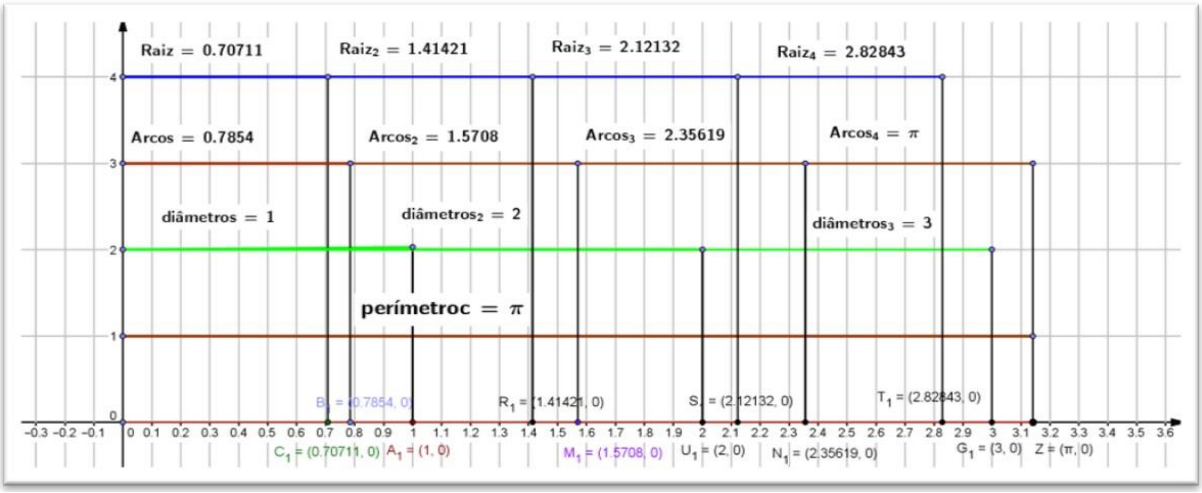

Source: Created by the author using GeoGebra (2021).

The search for the rationality of π, some historical approaches

1900. a. C.	Papiro Ahmes	Egípcio	28/34 x 3,1605
1600 a. C.	Tablet Susa	Babilônio	25/8 x 3,125
600 a. C.	Na Bíblia (Reis I, 7:23)	Feijão	3.00
500 a. C.	Bandhayana	Índia	3.09
250 a. C.	Arquimedes de Siracusa	Grego	3 10/71 e 3 1/7
150	Cláudio Ptolomeu	Greco egípcio	377/120- 3.14166...
263	Liu Hui	China	3.14159
263	Ventilador Wang	China	157/50 x 3,14
300	Chang Hong	China	101/2 x 3,1623
500	Para Chongzhi	China	3.1415926 - 3.14159
500	Aryabhata	Índia	3.1416
600	Brahma Gupta	Índia	101/2 x 3,1623
800	Al-Khurisimi	Perdido	3.1416
1220	Fibonacci	Italiano	3.141818
1400	Madhava	Índia	3.14159265359
1424	Al-Kashi	Perdido	2π = 6,2831853071

Fonte: Número π – Wikipédia, enciclopédia livre, (2022).

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2.5.3 CLASSICAL METHODS OF BABYLONIA (2000 B.C.)

Archaeologists have been working systematically in Mesopotamia since before the mid-19th century and have unearthed over half a million clay tablets, of which four hundred have been identified as strictly mathematical. Museums in Paris, Berlin, and London, as well as some universities like Yale, Columbia, and Pennsylvania, have excellent collections of these tablets, which vary in size and are approximately one and a half centimeters thick. They are often rounded in shape, with cuneiform inscriptions on only one of their faces, and sometimes on both.

The number of continuous fractions π inhabits approximately

$$\frac{6 * L}{2 * \pi * L} \approx 0,96 \leftrightarrow \frac{3}{\pi} \approx 0,96 \leftrightarrow (\pi \approx \frac{25}{8} \approx 3,125\ldots)$$

The number of the continuous fraction π, infinite periodic rational

$$\frac{6 * L}{2 * \pi * L} = 0.95492 \ldots \leftrightarrow \frac{3}{\pi} = 0.95492 \ldots \leftrightarrow (\pi = \frac{25.1327412287\ldots}{8.0000000000\ldots} = \pi)$$

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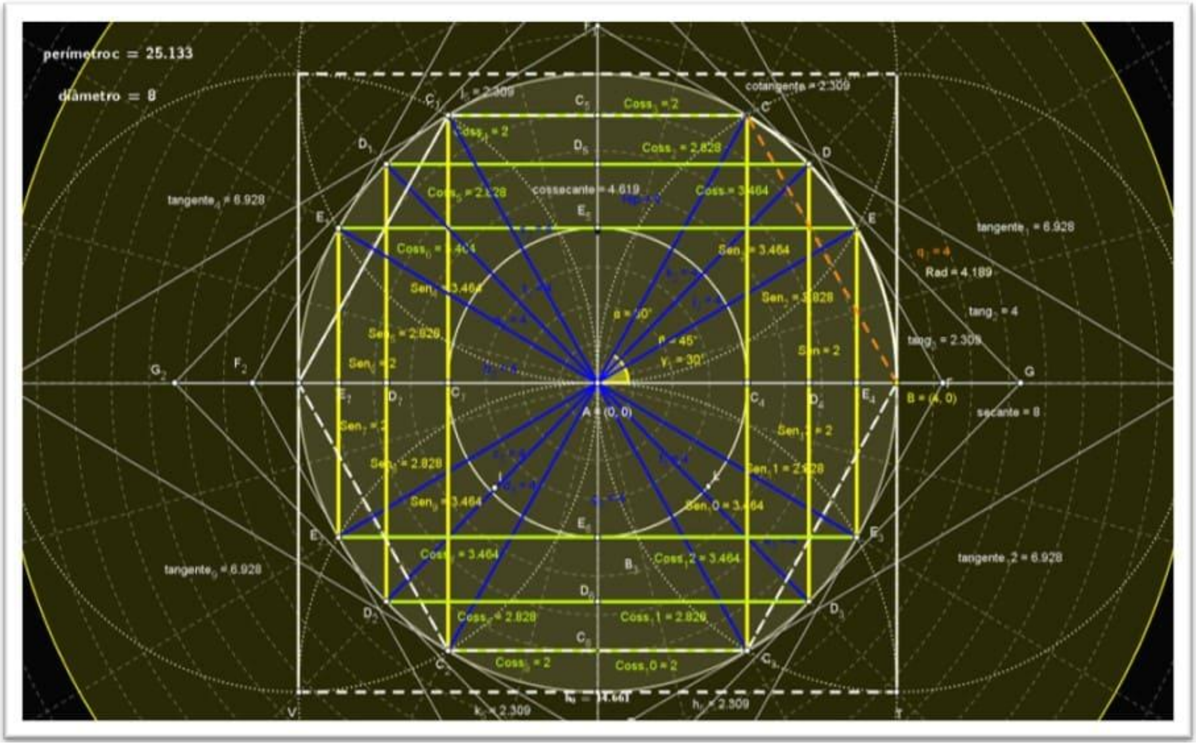
Figure 20. Demonstration of the division of the classical Babylonian method of the circumference



Círculos	{Perímetro c.30}	= 25.1327412287185230659355502185
Diâmetro	{Diâmetro. 30}	= 8.00000000000000000000000000000
Divisão	{Constante p,30}	= 3.14159265358938917154368732171

Métodos de cálculos por polígonos com valores aproximados irracionais

Polígonos	{lados,30}	≈ 4.00000000000000000000000000000
Círculos	{Perímetro c,30}	≈ 25.0000000000000000000000000000
Perímetros	{Lados* p.06}	≈ 24.0000000000000000000000000000
divisão	{Perímetro/ Lados*06}	≈1.00000000000000000000000000000

Métodos de cálculos por arcos de circunferência com valores exatos racionais

Arcos Ângulos	{Radianos,30}	= 4.18879020478585222872491642895
Círculos	{Perímetro c, 30}	=25.1327412287185230659355502185
Perímetros	{Rad. 30 *06}	=25.1327412287185230659355502185
Divisão racional	{Perímetro/ Lados*06}	=0.00000000000000000000000000000

Source: Created by the author using GeoGebra (2021).

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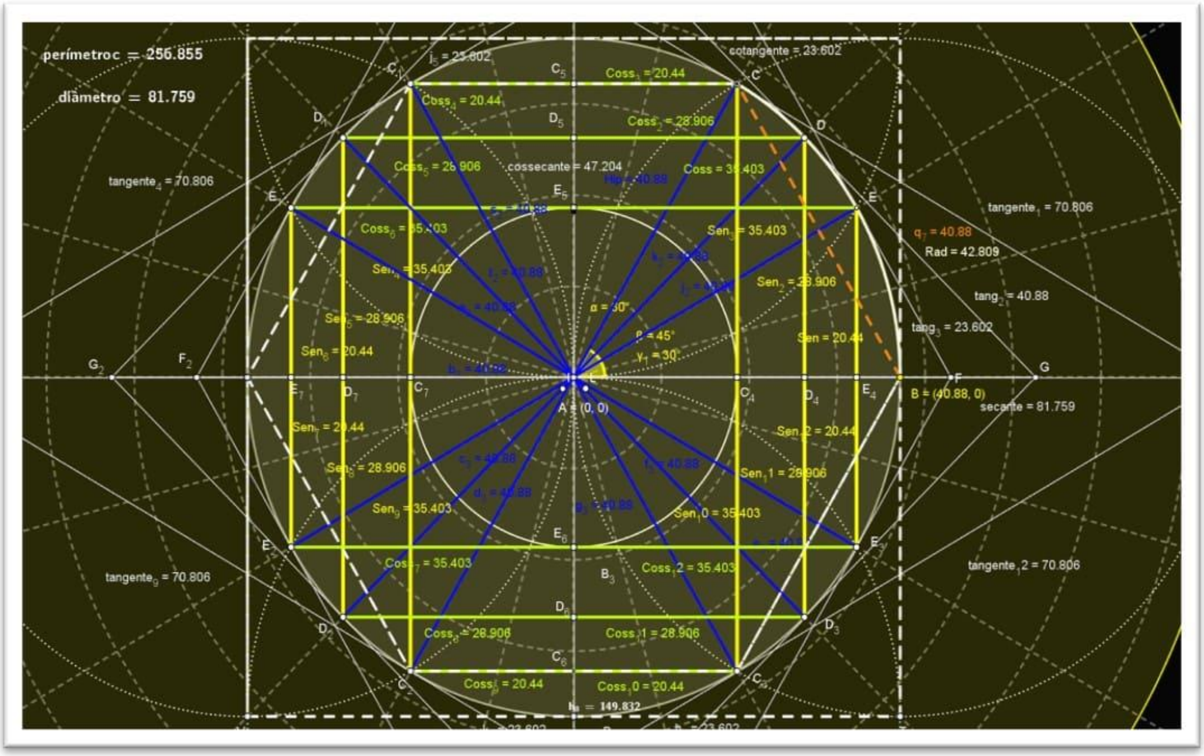
2.5.4 CLASSICAL METHODS OF THE RHIND PAPYRUS BY AHMES (1650 B.C.)

The Egyptian scribe Ahmes, 2000 B.C. Most of these problems come from measurement formulas necessary for calculating land areas and barn volumes. One of these records states that the area of a circle is equal to a square whose side is the diameter. $(\frac{8}{9})$. The continued fraction of π approximately inhabits:

$$\pi * r^2 = (3{,}160493\tfrac{8}{9}\ 2r) \rightarrow \pi \approx \frac{256}{81} \approx 3{,}160...$$

Figure 21. The classical methods of the Rhind papyrus with rational values



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=

Círculo	{Perímetro c.30}	= 256.854804585924561515851222382
Diâmetro	{Diâmetro. 30}	= 81.7594236135057363089761678535
Divisão	{Constante p,30}	= 3.14159265358938917154368732171

Métodos de cálculos por polígonos com valores aproximados irracionais

Polígonos	{lados,30}	≈ 40.00000000000000000000000000000
Círculos	{Perímetro c,30}	≈ 256.000000000000000000000000000
Perímetros	{Lados* p.06}	≈ 240.000000000000000000000000000
divisão	{Perímetro/ Lados*06}	≈16.00000000000000000000000000000

Métodos de cálculos por arcos de circunferência com valores exatos racionais

Arcos Ângulos	{Radianos,30}	= 42.8091340976533689754394955627
Círculos	{Perímetro c, 30}	=256.854804585924561515851222382
Perímetros	{Rad. 30 *06}	=256.854804585924561515851222382
Divisão racional	{Perímetro/ Lados*06}	=0.00000000000000000000000000000

Source: Created by the author using GeoGebra (2021).

2.5.5 ARCHIMEDES OF SYRACUSE (250 B.C.)

The Greek mathematician Archimedes of Syracuse (287-212 B.C.) discovered a highly efficient method for obtaining sequences of approximations of the constant π. In his work On the Measurement of the Circle, he developed a method of successive approximations for calculating the circumference of a circle.

He constructed regular polygons inscribed and circumscribed and divided the perimeter of each by the diameter of the circle, his studies with regular hexagons and doubling the number of sides of these polygons until reaching a polygon of 96 sides.

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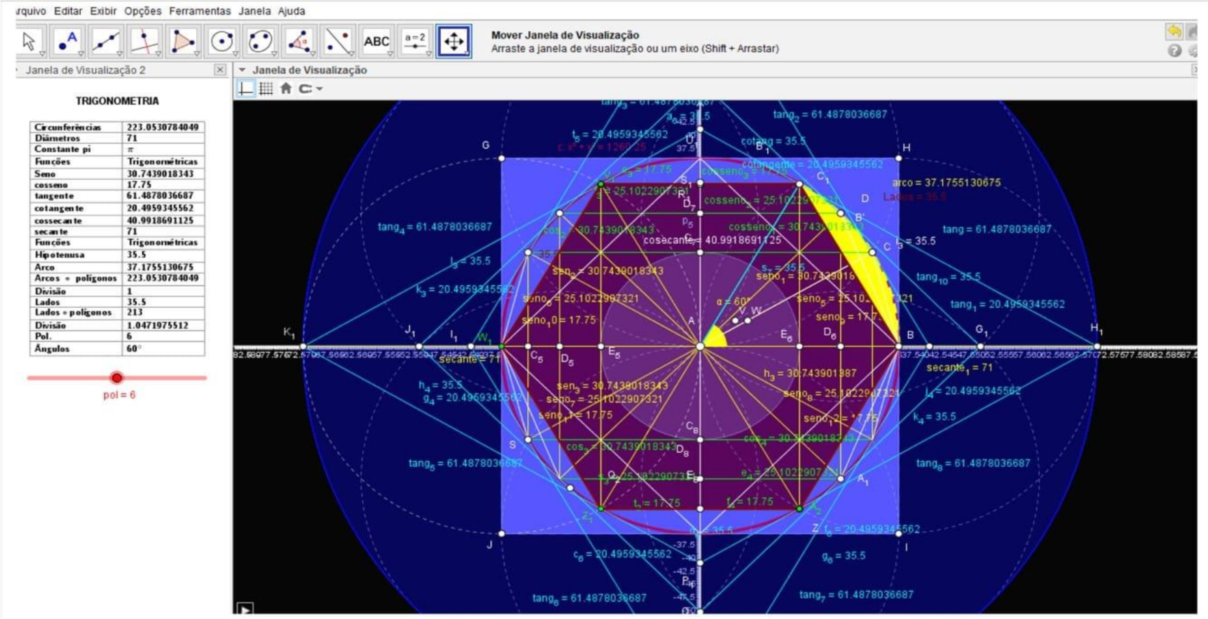
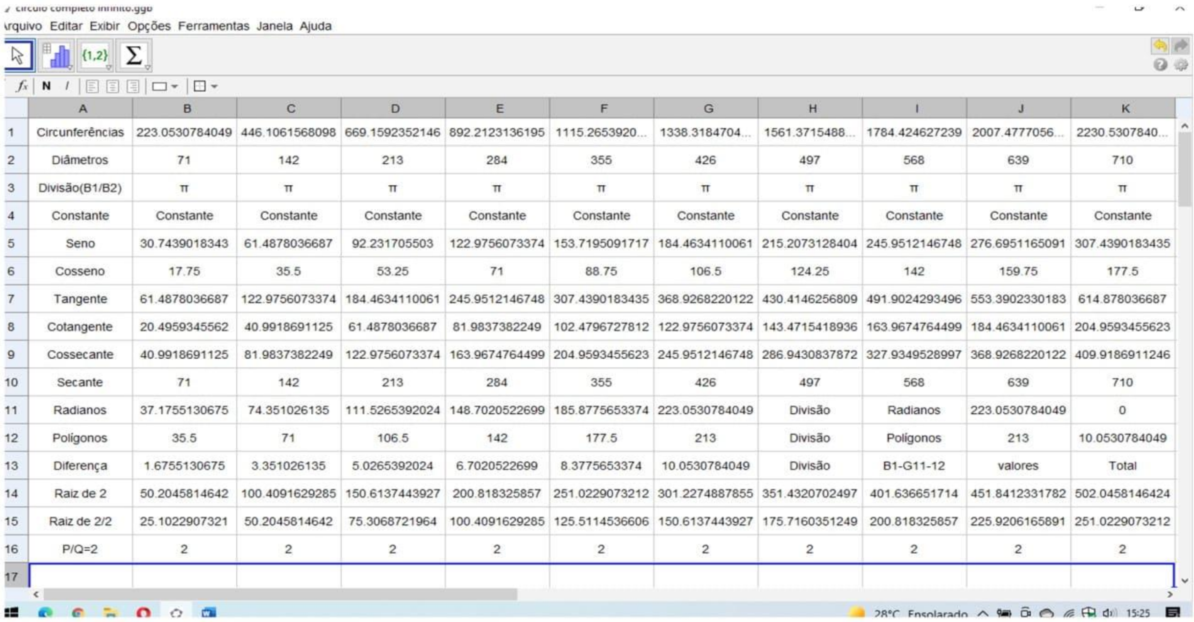
Figure 22. Demonstration of Archimedes' classical methods



Seja x um número racional. Assim, consideremos a fração contínua x, convergente e ilimitada, que a e b são números reais para b ≠ 0.

Circunferências =223.	**Diâmetros=71.**	**Divisões**
$3+\frac{10}{71}$	$\frac{223.131...}{71.025...}$	π=3,141...



	A	B	C	D	E	F	G	H	I	J	K
1	Circunferências	223.0530784049	446.1061568098	669.1592352146	892.2123136195	1115.2653920...	1338.3184704...	1561.3715488...	1784.424627239	2007.4777056...	2230.5307840...
2	Diâmetros	71	142	213	284	355	426	497	568	639	710
3	Divisão(B1/B2)	π	π	π	π	π	π	π	π	π	π
4	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante
5	Seno	30.7439018343	61.4878036687	92.231705503	122.9756073374	153.7195091717	184.4634110061	215.2073128404	245.9512146748	276.6951165091	307.4390183435
6	Cosseno	17.75	35.5	53.25	71	88.75	106.5	124.25	142	159.75	177.5
7	Tangente	61.4878036687	122.9756073374	184.4634110061	245.9512146748	307.4390183435	368.9268220122	430.4146256809	491.9024293496	553.3902330183	614.878036687
8	Cotangente	20.4959345562	40.9918691125	61.4878036687	81.9837382249	102.4796727812	122.9756073374	143.4715418936	163.9674764499	184.4634110061	204.9593455623
9	Cossecante	40.9918691125	81.9837382249	122.9756073374	163.9674764499	204.9593455623	245.9512146748	286.9430837872	327.9349528997	368.9268220122	409.9186911246
10	Secante	71	142	213	284	355	426	497	568	639	710
11	Radianos	37.1755130675	74.351026135	111.5265392024	148.7020522699	185.8775653374	223.0530784049	Divisão	Radianos	223.0530784049	0
12	Polígonos	35.5	71	106.5	142	177.5	213	Divisão	Polígonos	213	10.0530784049
13	Diferença	1.6755130675	3.351026135	5.0265392024	6.7020522699	8.3775653374	10.0530784049	Divisão	B1-G11-12	valores	Total
14	Raiz de 2	50.2045814642	100.4091629285	150.6137443927	200.818325857	251.0229073212	301.2274887855	351.4320702497	401.636651714	451.8412331782	502.0458146424
15	Raiz de 2/2	25.1022907321	50.2045814642	75.3068721964	100.4091629285	125.5114536606	150.6137443927	175.7160351249	200.818325857	225.9206165891	251.0229073212
16	P/Q=2	2	2	2	2	2	2	2	2	2	2
17											

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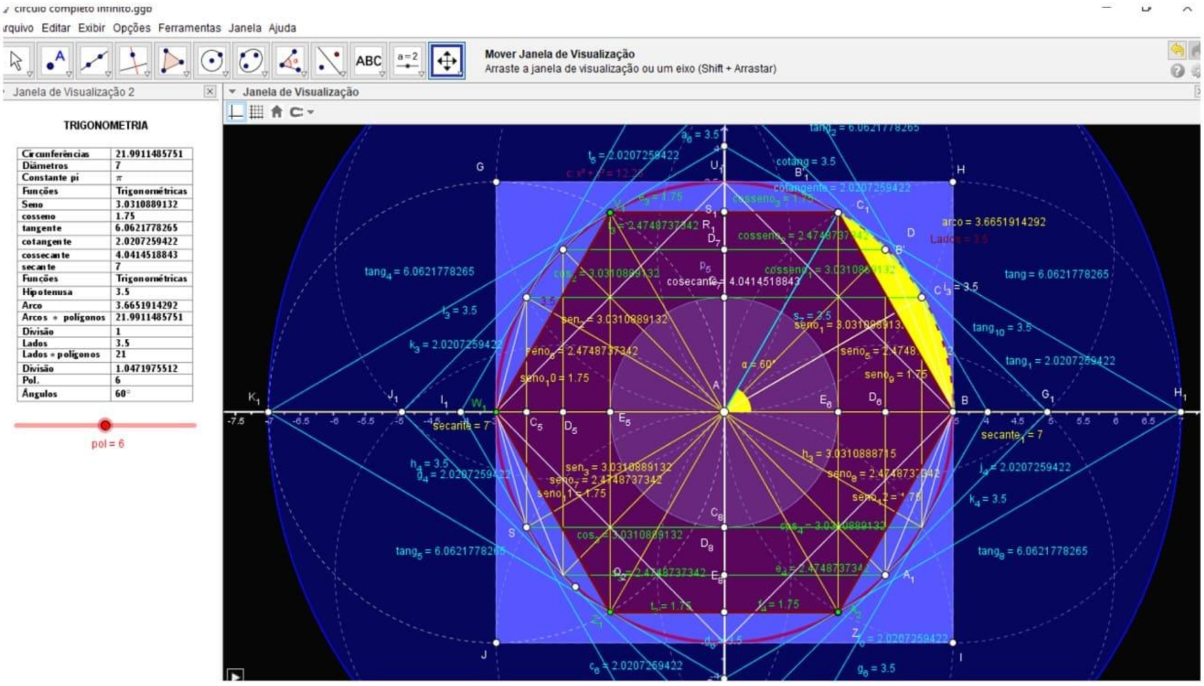
Figure 23. Archimedes' classical methods with rational values



Seja π um número racional. Assim, consideremos a fração π, convergente e ilimitada, de que a e b são números reais para $b \neq 0$.

Circunferências = 22.	Diâmetros=7.	Divisões
$3 + \frac{1}{7}$	$\frac{22.02\ldots}{7.01\ldots}$	π=3,141...

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	A	B	C	D	E	F	G	H	I	J	K
1	Circunferências	21.9911485751	43.9822971503	65.9734457254	87.9645943005	109.9557428756	131.9468914508	153.9380400259	175.929188601	197.9203371762	219.9114857513
2	Diâmetros	7	14	21	28	35	42	49	56	63	70
3	Divisão(B1/B2)	π	π	π	π	π	π	π	π	π	π
4	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante
5	Seno	3.0310889132	6.0621778265	9.0932667397	12.124355653	15.1554445662	18.1865334795	21.2176223927	24.248711306	27.2798002192	30.3108891325
6	Cosseno	1.75	3.5	5.25	7	8.75	10.5	12.25	14	15.75	17.5
7	Tangente	6.0621778265	12.124355653	18.1865334795	24.248711306	30.3108891325	36.3730669589	42.4352447854	48.4974226119	54.5596004384	60.6217782649
8	Cotangente	2.0207259422	4.0414518843	6.0621778265	8.0829037687	10.1036297108	12.124355653	14.1450815951	16.1658075373	18.1865334795	20.2072594216
9	Cossecante	4.0414518843	8.0829037687	12.124355653	16.1658075373	20.2072594216	24.248711306	28.2901631903	32.3316150746	36.3730669589	40.4145188433
10	Secante	7	14	21	28	35	42	49	56	63	70
11	Radianos	3.6651914292	7.3303828584	10.9955742876	14.6607657168	18.3259571459	21.9911485751	Divisão	Radianos	21.9911485751	0
12	Polígonos	3.5	7	10.5	14	17.5	21	Divisão	Polígonos	21	0.9911485751
13	Diferença	0.1651914292	0.3303828584	0.4955742876	0.6607657168	0.8259571459	0.9911485751	Divisão	B1-G11-12	valores	Total
14	Raiz de 2	4.9497474683	9.8994949366	14.8492424049	19.7989898732	24.7487373415	29.6984848098	34.6482322781	39.5979797464	44.5477272148	49.4974746831
15	Raiz de 2/2	2.4748737342	4.9497474683	7.4246212025	9.8994949366	12.3743686708	14.8492424049	17.3241161391	19.7989898732	22.2738636074	24.7487373415
16	P/Q=2	2	2	2	2	2	2	2	2	2	2
17											

Source: Created by the author using GeoGebra (2021).

2.6 ARCS AND ANGLES OF A CIRCUMFERENCE (DEGREE AND RADIANS)

The circumference in which two points A and B are taken. The circumference will be divided into two parts called arcs. Points A and B are the endpoints of these arcs. When A and B coincide, one of these arcs is called null and the other, an arc of one turn; we will say that the null arc measures 0º and the arc of one turn measures 360º. This way: 1 grau(1°) $=\frac{1}{360}$ of the arc of one turn. As submultiples of the degree, we have: 1 minute (1’) $=\frac{1}{60}$, of the degree, or 60 minutes = 1 degree (60’=1°), and 1 second (1”) $=\frac{1}{60}$ of the minute, or 60 seconds = 1 minute (60” = 1’).

A radian is the measure of an arc that corresponds to the same length as the radius (r) of the circumference in relation to the central angle.

An arc of one radian (1 rad) is one whose length is equal to the radius of the circumference. C = 2*π*r – 360º – 2 π radianos (NETO, 1978).

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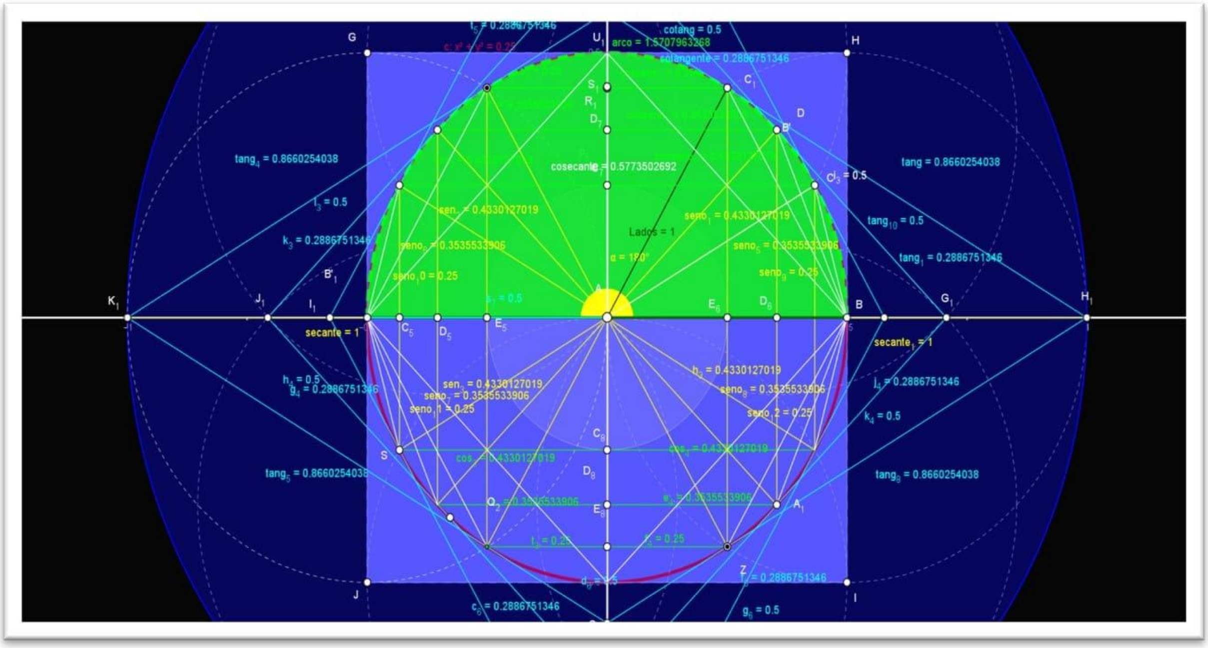
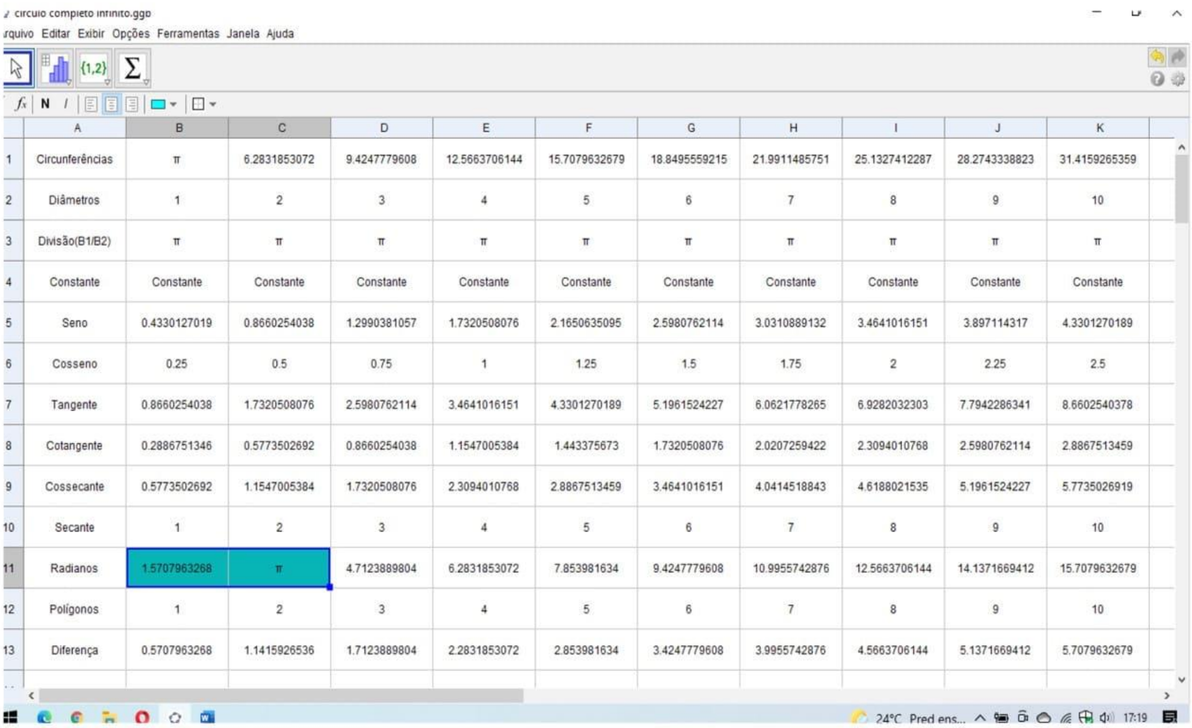
Figure 24. Demonstration of the dynamic trigonometric circle infinite rationals





	A	B	C	D	E	F	G	H	I	J	K
1	Circunferências	π	6.2831853072	9.4247779608	12.5663706144	15.7079632679	18.8495559215	21.9911485751	25.1327412287	28.2743338823	31.4159265359
2	Diâmetros	1	2	3	4	5	6	7	8	9	10
3	Divisão(B1/B2)	π	π	π	π	π	π	π	π	π	π
4	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante
5	Seno	0.4330127019	0.8660254038	1.2990381057	1.7320508076	2.1650635095	2.5980762114	3.0310889132	3.4641016151	3.897114317	4.3301270189
6	Cosseno	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5
7	Tangente	0.8660254038	1.7320508076	2.5980762114	3.4641016151	4.3301270189	5.1961524227	6.0621778265	6.9282032303	7.7942286341	8.6602540378
8	Cotangente	0.2886751346	0.5773502692	0.8660254038	1.1547005384	1.443375673	1.7320508076	2.0207259422	2.3094010768	2.5980762114	2.8867513459
9	Cossecante	0.5773502692	1.1547005384	1.7320508076	2.3094010768	2.8867513459	3.4641016151	4.0414518843	4.6188021535	5.1961524227	5.7735026919
10	Secante	1	2	3	4	5	6	7	8	9	10
11	Radianos	1.5707963268	π	4.7123889804	6.2831853072	7.853981634	9.4247779608	10.9955742876	12.5663706144	14.1371669412	15.7079632679
12	Polígonos	1	2	3	4	5	6	7	8	9	10
13	Diferença	0.5707963268	1.1415926536	1.7123889804	2.2831853072	2.853981634	3.4247779608	3.9955742876	4.5663706144	5.1371669412	5.7079632679

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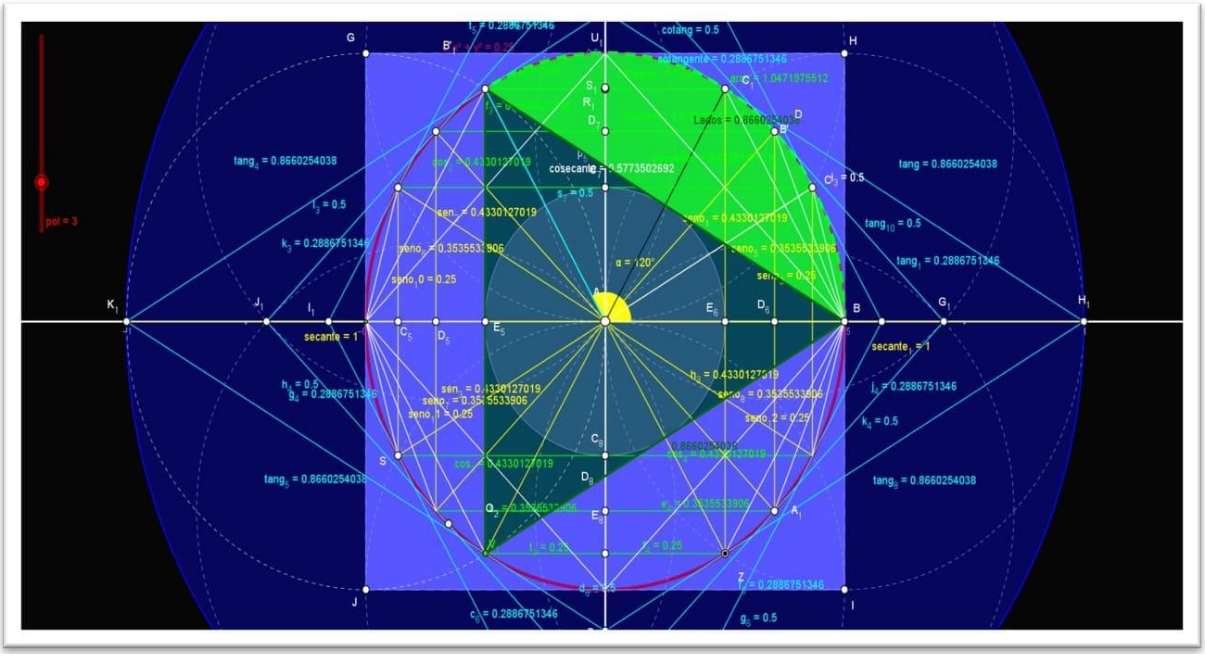
Figure 25. Demonstration between circles and their diameters rational values



circulo completo infinito.ggb

Arquivo Editar Exibir Opções Ferramentas Janela Ajuda

	A	B	C	D	E	F	G	H	I	J	K
1	Circunferências	π	6.2831853072	9.4247779608	12.5663706144	15.7079632679	18.8495559215	21.9911485751	25.1327412287	28.2743338823	31.4159265359
2	Diâmetros	1	2	3	4	5	6	7	8	9	10
3	Divisão(B1/B2)	π	π	π	π	π	π	π	π	π	π
4	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante
5	Seno	0.4330127019	0.8660254038	1.2990381057	1.7320508076	2.1650635095	2.5980762114	3.0310889132	3.4641016151	3.897114317	4.3301270189
6	Cosseno	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5
7	Tangente	0.8660254038	1.7320508076	2.5980762114	3.4641016151	4.3301270189	5.1961524227	6.0621778265	6.9282032303	7.7942286341	8.6602540378
8	Cotangente	0.2886751346	0.5773502692	0.8660254038	1.1547005384	1.443375673	1.7320508076	2.0207259422	2.3094010768	2.5980762114	2.8867513459
9	Cossecante	0.5773502692	1.1547005384	1.7320508076	2.3094010768	2.8867513459	3.4641016151	4.0414518843	4.6188021535	5.1961524227	5.7735026919
10	Secante	1	2	3	4	5	6	7	8	9	10
11	Radianos	1.0471975512	2.0943951024	3.1415926536	4.1887902048	5.235987756	6.2831853072	7.3303828584	8.3775804096	9.4247779608	10.471975512
12	Polígonos	0.8660254038	1.7320508076	2.5980762114	3.4641016151	4.3301270189	5.1961524227	6.0621778265	6.9282032303	7.7942286341	8.6602540378
13	Diferença	0.1811721474	0.3623442948	0.5435164422	0.7246885896	0.9058607371	1.0870328845	1.2682050319	1.4493771793	1.6305493267	1.8117214741

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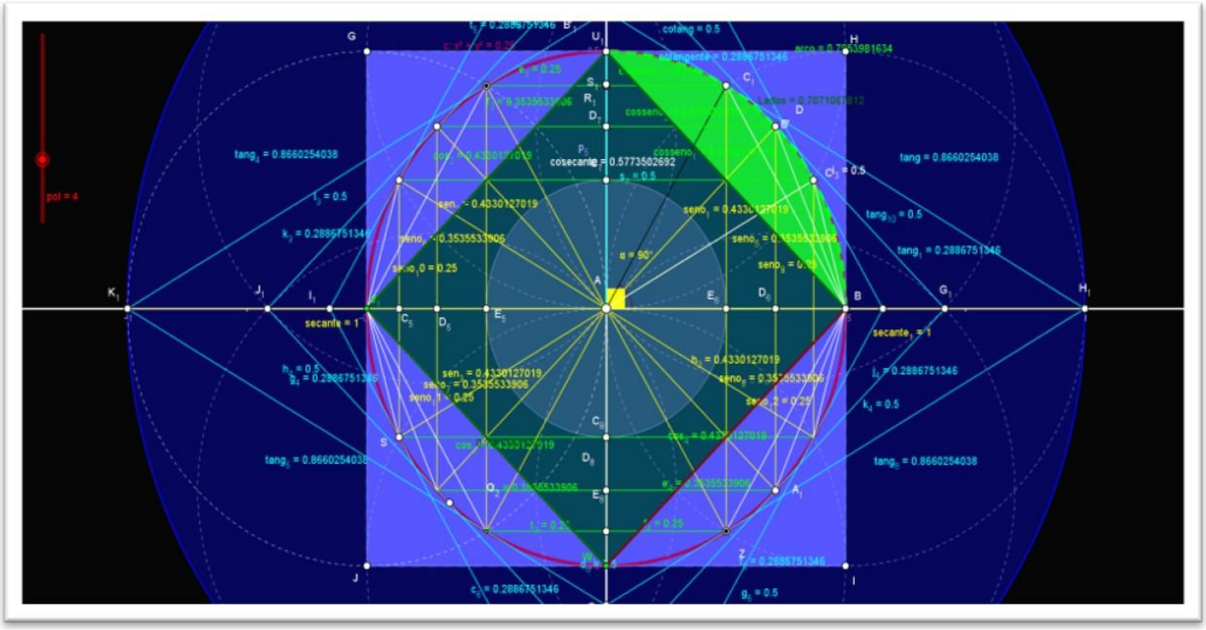
Figure 26. Demonstration between circles and their diameters rational values



	A	B	C	D	E	F	G	H	I	J	K
1	Circunferências	π	6.2831853072	9.4247779608	12.5663706144	15.7079632679	18.8495559215	21.9911485751	25.1327412287	28.2743338823	31.4159265359
2	Diâmetros	1	2	3	4	5	6	7	8	9	10
3	Divisão(B1/B2)	π	π	π	π	π	π	π	π	π	π
4	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante
5	Seno	0.4330127019	0.8660254038	1.2990381057	1.7320508076	2.1650635095	2.5980762114	3.0310889132	3.4641016151	3.897114317	4.3301270189
6	Cosseno	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5
7	Tangente	0.8660254038	1.7320508076	2.5980762114	3.4641016151	4.3301270189	5.1961524227	6.0621778265	6.9282032303	7.7942286341	8.6602540378
8	Cotangente	0.2886751346	0.5773502692	0.8660254038	1.1547005384	1.443375673	1.7320508076	2.0207259422	2.3094010768	2.5980762114	2.8867513459
9	Cossecante	0.5773502692	1.1547005384	1.7320508076	2.3094010768	2.8867513459	3.4641016151	4.0414518843	4.6188021535	5.1961524227	5.7735026919
10	Secante	1	2	3	4	5	6	7	8	9	10
11	Radianos	0.7853981634	1.5707963268	2.3561944902	π	3.926990817	4.7123889804	5.4977871438	6.2831853072	7.0685834706	7.853981634
12	Polígonos	0.7071067812	1.4142135624	2.1213203436	2.8284271247	3.5355339059	4.2426406871	4.9497474683	5.6568542495	6.3639610307	7.0710678119
13	Diferença	0.0782913822	0.1565827644	0.2348741466	0.3131655288	0.3914569111	0.4697482933	0.5480396755	0.6263310577	0.7046224399	0.7829138221

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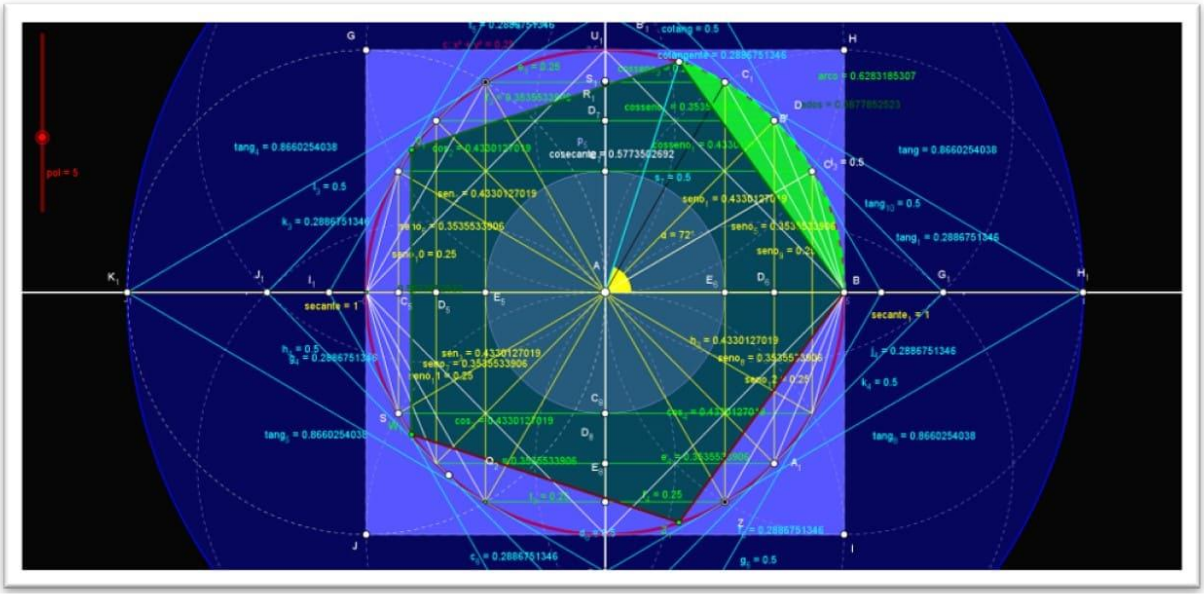
Figure 27. Demonstration between circles and their diameters rational values



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Arquivo Editar Exibir Opções Ferramentas Janela Ajuda

	A	B	C	D	E	F	G	H	I	J	K
1	Circunferências	π	6.2831853072	9.4247779608	12.5663706144	15.7079632679	18.8495559215	21.9911485751	25.1327412287	28.2743338823	31.4159265359
2	Diâmetros	1	2	3	4	5	6	7	8	9	10
3	Divisão(B1/B2)	π	π	π	π	π	π	π	π	π	π
4	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante
5	Seno	0.4330127019	0.8660254038	1.2990381057	1.7320508076	2.1650635095	2.5980762114	3.0310889132	3.4641016151	3.897114317	4.3301270189
6	Cosseno	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5
7	Tangente	0.8660254038	1.7320508076	2.5980762114	3.4641016151	4.3301270189	5.1961524227	6.0621778265	6.9282032303	7.7942286341	8.6602540378
8	Cotangente	0.2886751346	0.5773502692	0.8660254038	1.1547005384	1.443375673	1.7320508076	2.0207259422	2.3094010768	2.5980762114	2.8867513459
9	Cossecante	0.5773502692	1.1547005384	1.7320508076	2.3094010768	2.8867513459	3.4641016151	4.0414518843	4.6188021535	5.1961524227	5.7735026919
10	Secante	1	2	3	4	5	6	7	8	9	10
11	Radianos	0.6283185307	1.2566370614	1.8849555922	2.5132741229	π	3.7699111843	4.398229715	5.0265482457	5.6548667765	6.2831853072
12	Polígonos	0.5877852523	1.1755705046	1.7633557569	2.3511410092	2.9389262615	3.5267115138	4.114496766	4.7022820183	5.2900672706	5.8778525229
13	Diferença	0.0405332784	0.0810665569	0.1215998353	0.1621331137	0.2026663921	0.2431996706	0.283732949	0.3242662274	0.3647995058	0.4053327843

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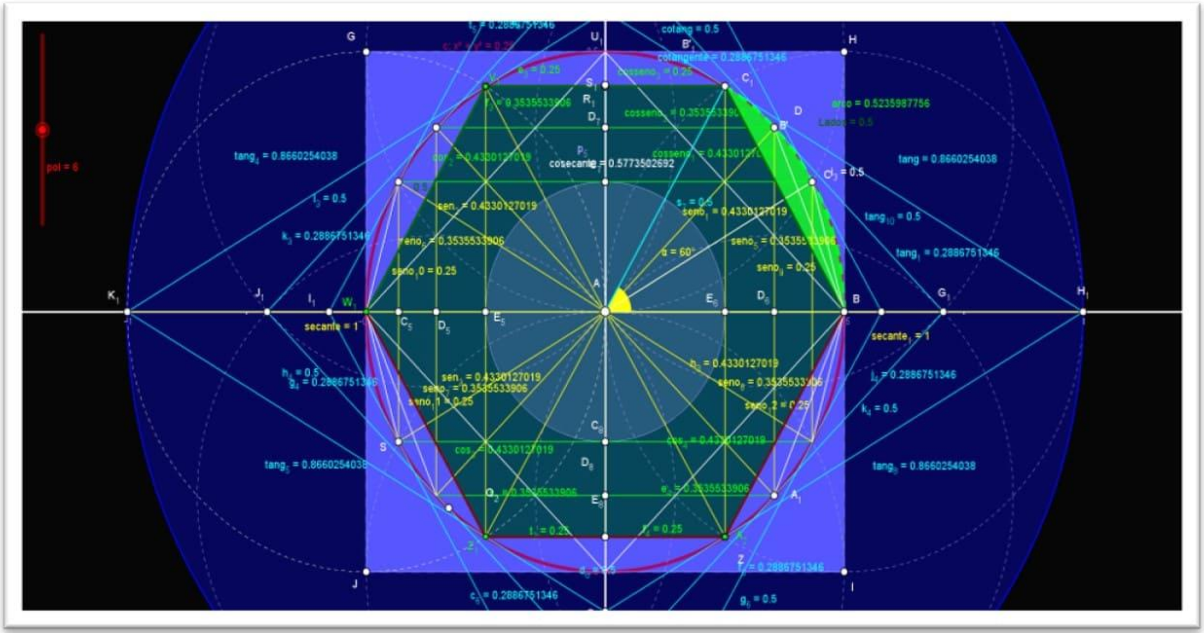
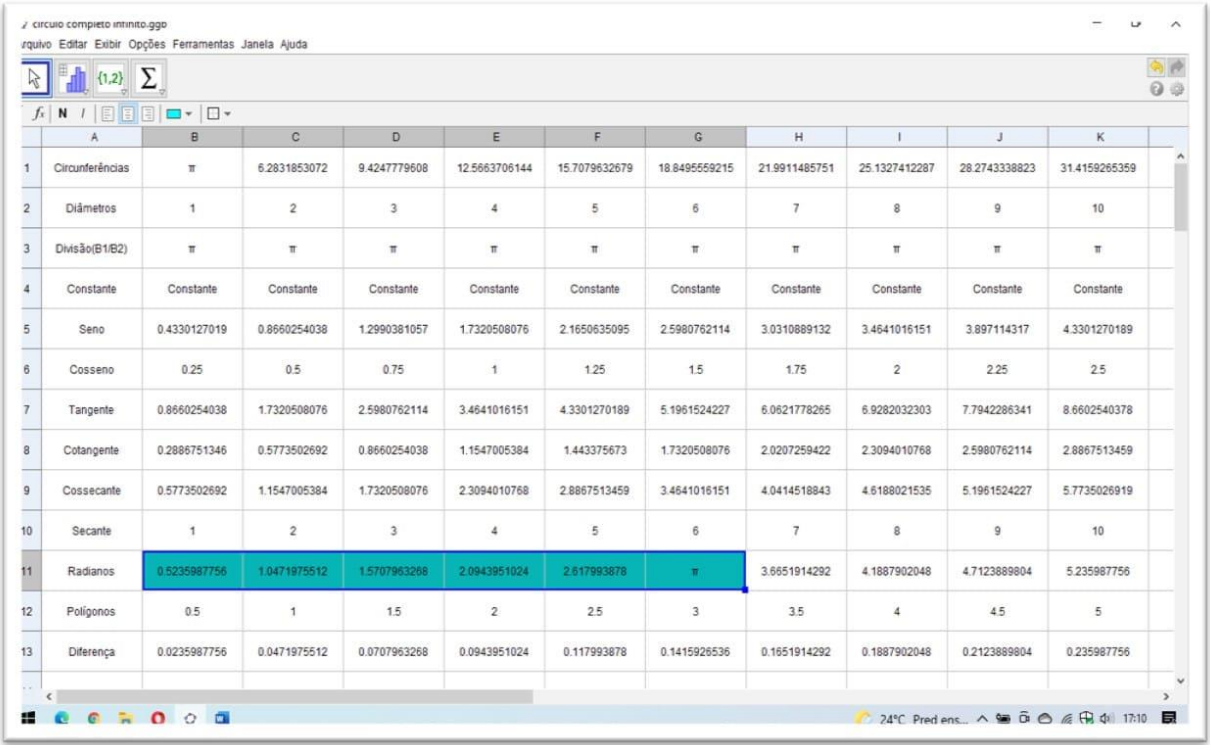
Figure 28. Demonstration between circles and their diameters rational values





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Arquivo Editar Exibir Opções Ferramentas Janela Ajuda

	A	B	C	D	E	F	G	H	I	J	K
1	Circunferências	π	6.2831853072	9.4247779608	12.5663706144	15.7079632679	18.8495559215	21.9911485751	25.1327412287	28.2743338823	31.4159265359
2	Diâmetros	1	2	3	4	5	6	7	8	9	10
3	Divisão(B1/B2)	π	π	π	π	π	π	π	π	π	π
4	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante
5	Seno	0.4330127019	0.8660254038	1.2990381057	1.7320508076	2.1650635095	2.5980762114	3.0310889132	3.4641016151	3.897114317	4.3301270189
6	Cosseno	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5
7	Tangente	0.8660254038	1.7320508076	2.5980762114	3.4641016151	4.3301270189	5.1961524227	6.0621778265	6.9282032303	7.7942286341	8.6602540378
8	Cotangente	0.2886751346	0.5773502692	0.8660254038	1.1547005384	1.443375673	1.7320508076	2.0207259422	2.3094010768	2.5980762114	2.8867513459
9	Cossecante	0.5773502692	1.1547005384	1.7320508076	2.3094010768	2.8867513459	3.4641016151	4.0414518843	4.6188021535	5.1961524227	5.7735026919
10	Secante	1	2	3	4	5	6	7	8	9	10
11	Radianos	0.5235987756	1.0471975512	1.5707963268	2.0943951024	2.617993878	π	3.6651914292	4.1887902048	4.7123889804	5.235987756
12	Polígonos	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
13	Diferença	0.0235987756	0.0471975512	0.0707963268	0.0943951024	0.117993878	0.1415926536	0.1651914292	0.1887902048	0.2123889804	0.235987756

Source: Created by the author using GeoGebra (2021).

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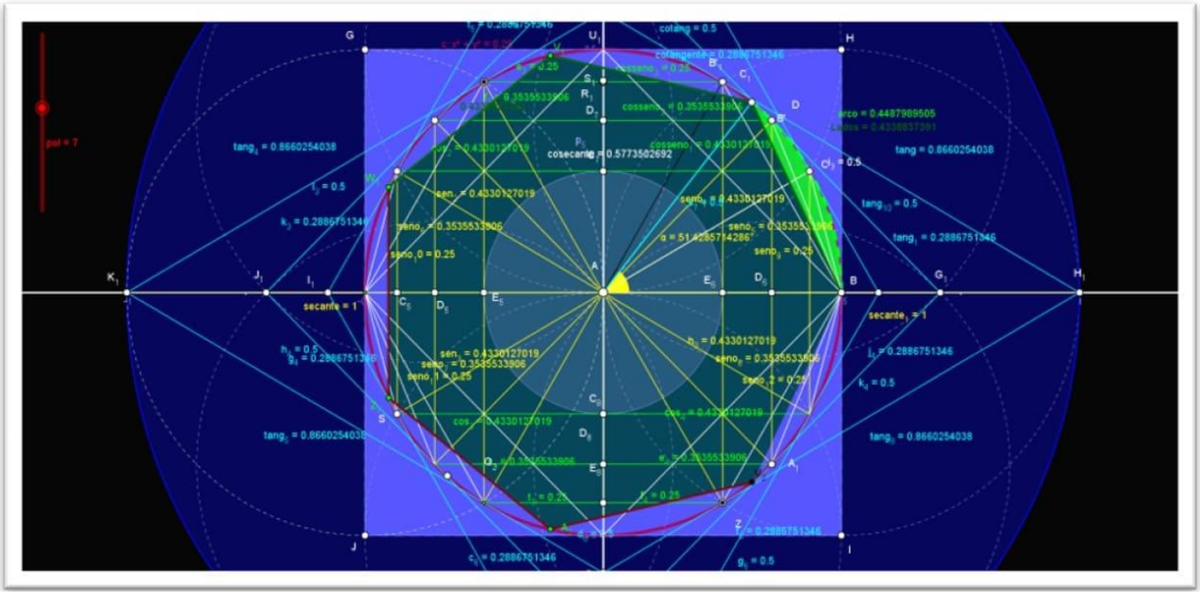
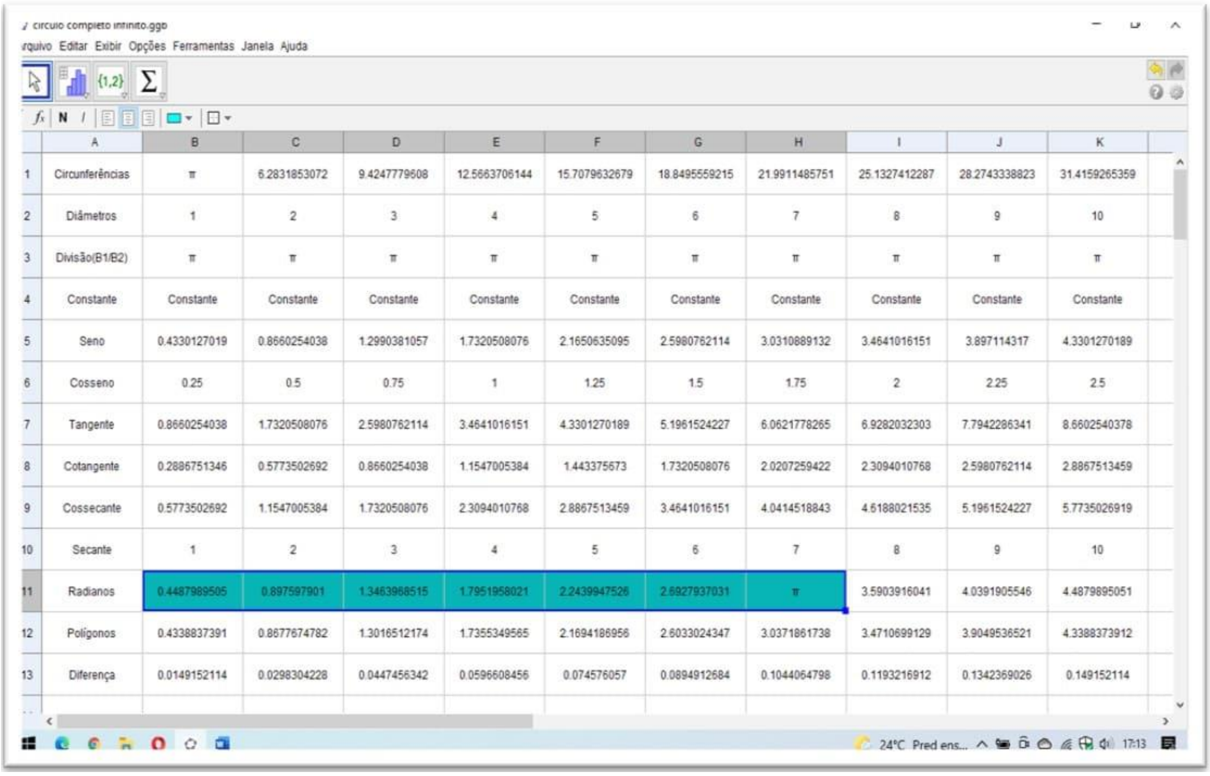
Figure 29. Demonstration between circles and their diameters rational values





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Arquivo Editar Exibir Opções Ferramentas Janela Ajuda

	A	B	C	D	E	F	G	H	I	J	K
1	Circunferências	π	6.2831853072	9.4247779608	12.5663706144	15.7079632679	18.8495559215	21.9911485751	25.1327412287	28.2743338823	31.4159265359
2	Diâmetros	1	2	3	4	5	6	7	8	9	10
3	Divisão(B1/B2)	π	π	π	π	π	π	π	π	π	π
4	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante
5	Seno	0.4330127019	0.8660254038	1.2990381057	1.7320508076	2.1650635095	2.5980762114	3.0310889132	3.4641016151	3.897114317	4.3301270189
6	Cosseno	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5
7	Tangente	0.8660254038	1.7320508076	2.5980762114	3.4641016151	4.3301270189	5.1961524227	6.0621778265	6.9282032303	7.7942286341	8.6602540378
8	Cotangente	0.2886751346	0.5773502692	0.8660254038	1.1547005384	1.443375673	1.7320508076	2.0207259422	2.3094010768	2.5980762114	2.8867513459
9	Cossecante	0.5773502692	1.1547005384	1.7320508076	2.3094010768	2.8867513459	3.4641016151	4.0414518843	4.6188021535	5.1961524227	5.7735026919
10	Secante	1	2	3	4	5	6	7	8	9	10
11	Radianos	0.4487989505	0.897597901	1.3463968515	1.7951958021	2.2439947526	2.6927937031	π	3.5903916041	4.0391905546	4.4879895051
12	Polígonos	0.4338837391	0.8677674782	1.3016512174	1.7355349565	2.1694186956	2.6033024347	3.0371861738	3.4710699129	3.9049536521	4.3388373912
13	Diferença	0.0149152114	0.0298304228	0.0447456342	0.0596608456	0.074576057	0.0894912684	0.1044064798	0.1193216912	0.1342369026	0.149152114

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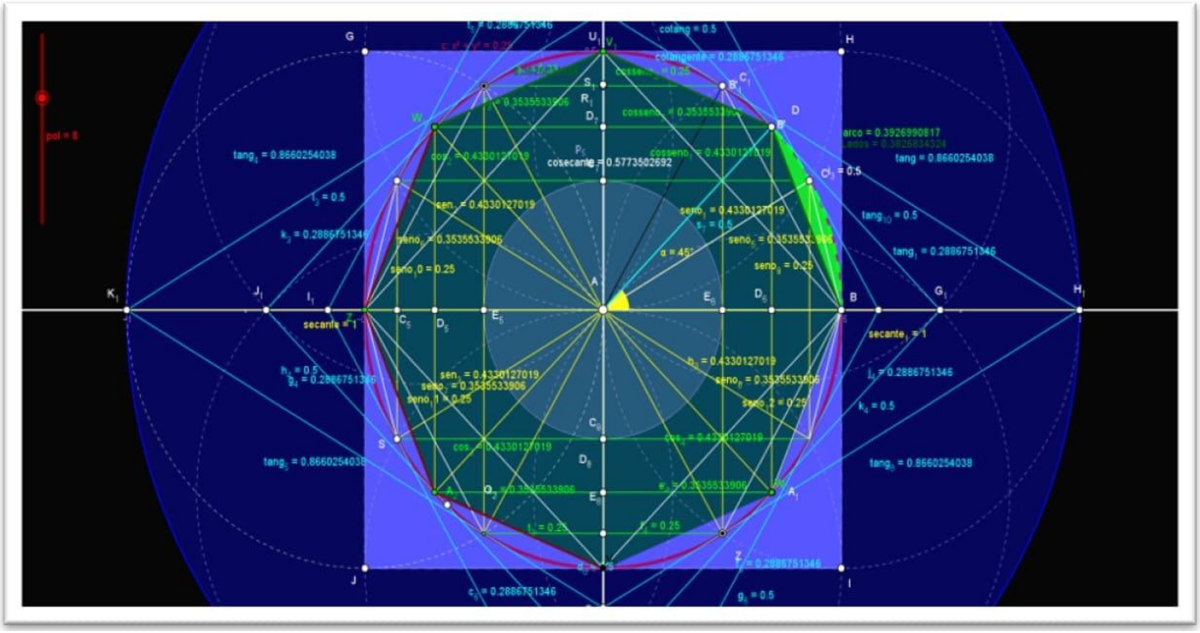
Figure 30. Demonstration between circles and their diameters rational values



	A	B	C	D	E	F	G	H	I	J	K
1	Circunferências	π	6.2831853072	9.4247779608	12.5663706144	15.7079632679	18.8495559215	21.9911485751	25.1327412287	28.2743338823	31.4159265359
2	Diâmetros	1	2	3	4	5	6	7	8	9	10
3	Divisão(B1/B2)	π	π	π	π	π	π	π	π	π	π
4	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante
5	Seno	0.4330127019	0.8660254038	1.2990381057	1.7320508076	2.1650635095	2.5980762114	3.0310889132	3.4641016151	3.897114317	4.3301270189
6	Cosseno	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5
7	Tangente	0.8660254038	1.7320508076	2.5980762114	3.4641016151	4.3301270189	5.1961524227	6.0621778265	6.9282032303	7.7942286341	8.6602540378
8	Cotangente	0.2886751346	0.5773502692	0.8660254038	1.1547005384	1.443375673	1.7320508076	2.0207259422	2.3094010768	2.5980762114	2.8867513459
9	Cossecante	0.5773502692	1.1547005384	1.7320508076	2.3094010768	2.8867513459	3.4641016151	4.0414518843	4.6188021535	5.1961524227	5.7735026919
10	Secante	1	2	3	4	5	6	7	8	9	10
11	Radianos	0.3926990817	0.7853981634	1.1780972451	1.5707963268	1.9634954085	2.3561944902	2.7488935719	π	3.5342917353	3.926990817
12	Polígonos	0.3826834324	0.7653668647	1.1480502971	1.5307337295	1.9134171618	2.2961005942	2.6787840266	3.0614674589	3.4441508913	3.8268343237
13	Diferença	0.0100156493	0.0200312987	0.030046948	0.0400625973	0.0500782467	0.060093896	0.0701095453	0.0801251947	0.090140844	0.1001564933

Source: Created by the author using GeoGebra (2021).[/caption]

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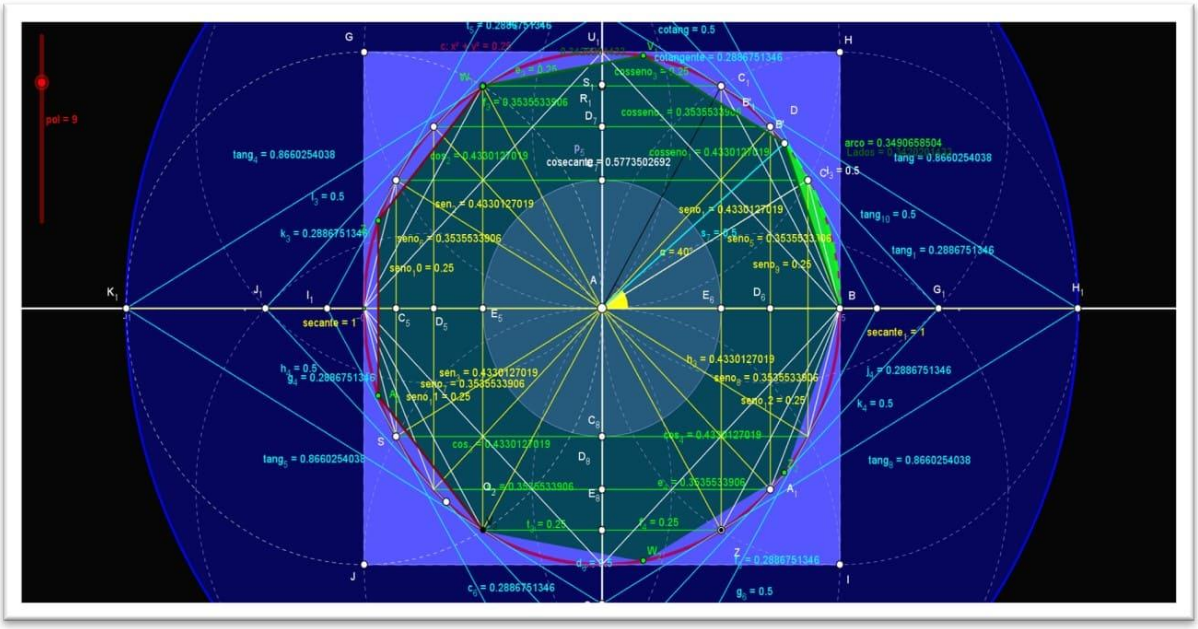
Figure 31. Demonstration between circles and their diameters rational values



	A	B	C	D	E	F	G	H	I	J	K
1	Circunferências	π	6.2831853072	9.4247779608	12.5663706144	15.7079632679	18.8495559215	21.9911485751	25.1327412287	28.2743338823	31.4159265359
2	Diâmetros	1	2	3	4	5	6	7	8	9	10
3	Divisão(B1/B2)	π	π	π	π	π	π	π	π	π	π
4	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante
5	Seno	0.4330127019	0.8660254038	1.2990381057	1.7320508076	2.1650635095	2.5980762114	3.0310889132	3.4641016151	3.897114317	4.3301270189
6	Cosseno	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5
7	Tangente	0.8660254038	1.7320508076	2.5980762114	3.4641016151	4.3301270189	5.1961524227	6.0621778265	6.9282032303	7.7942286341	8.6602540378
8	Cotangente	0.2886751346	0.5773502692	0.8660254038	1.1547005384	1.443375673	1.7320508076	2.0207259422	2.3094010768	2.5980762114	2.8867513459
9	Cossecante	0.5773502692	1.1547005384	1.7320508076	2.3094010768	2.8867513459	3.4641016151	4.0414518843	4.6188021535	5.1961524227	5.7735026919
10	Secante	1	2	3	4	5	6	7	8	9	10
11	Radianos	0.3490658504	0.6981317008	1.0471975512	1.3962634016	1.745329252	2.0943951024	2.4434609528	2.7925268032	π	3.490658504
12	Polígonos	0.3420201433	0.6840402867	1.02606043	1.3680805733	1.7101007166	2.05212086	2.3941410033	2.7361611466	3.0781812899	3.4202014333
13	Diferença	0.0070457071	0.0140914141	0.0211371212	0.0281828283	0.0352285354	0.0422742424	0.0493199495	0.0563656566	0.0634113637	0.0704570707

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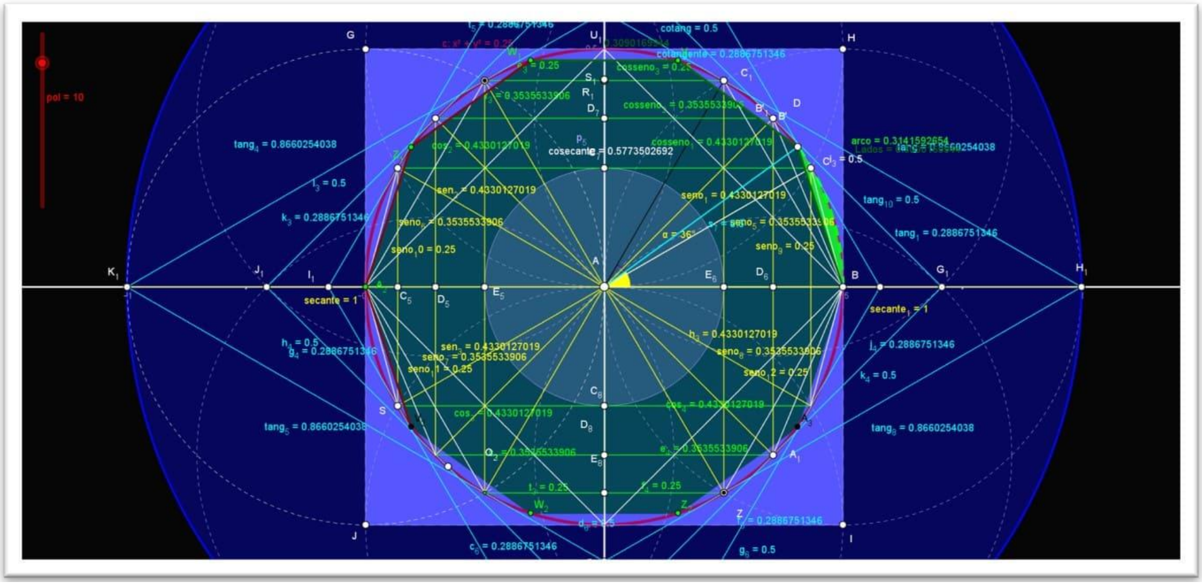
Figura 32. Demonstração entre circunferências e seus diâmetros valores racionais



	A	B	C	D	E	F	G	H	I	J	K
1	Circunferências	π	6.2831853072	9.4247779608	12.5663706144	15.7079632679	18.8495559215	21.9911485751	25.1327412287	28.2743338823	31.4159265359
2	Diâmetros	1	2	3	4	5	6	7	8	9	10
3	Divisão(B1/B2)	π	π	π	π	π	π	π	π	π	π
4	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante
5	Seno	0.4330127019	0.8660254038	1.2990381057	1.7320508076	2.1650635095	2.5980762114	3.0310889132	3.4641016151	3.897114317	4.3301270189
6	Cosseno	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5
7	Tangente	0.8660254038	1.7320508076	2.5980762114	3.4641016151	4.3301270189	5.1961524227	6.0621778265	6.9282032303	7.7942286341	8.6602540378
8	Cotangente	0.2886751346	0.5773502692	0.8660254038	1.1547005384	1.443375673	1.7320508076	2.0207259422	2.3094010768	2.5980762114	2.8867513459
9	Cossecante	0.5773502692	1.1547005384	1.7320508076	2.3094010768	2.8867513459	3.4641016151	4.0414518843	4.6188021535	5.1961524227	5.7735026919
10	Secante	1	2	3	4	5	6	7	8	9	10
11	Radianos	0.3141592654	0.6283185307	0.9424777961	1.2566370614	1.5707963268	1.8849555922	2.1991148575	2.5132741229	2.8274333882	π
12	Polígonos	0.3090169944	0.6180339887	0.9270509831	1.2360679775	1.5450849719	1.8541019662	2.1631189606	2.472135955	2.7811529494	3.0901699437
13	Diferença	0.005142271	0.010284542	0.015426813	0.0205690839	0.0257113549	0.0308536259	0.0359958969	0.0411381679	0.0462804389	0.0514227098

Source: Created by the author using GeoGebra (2021).

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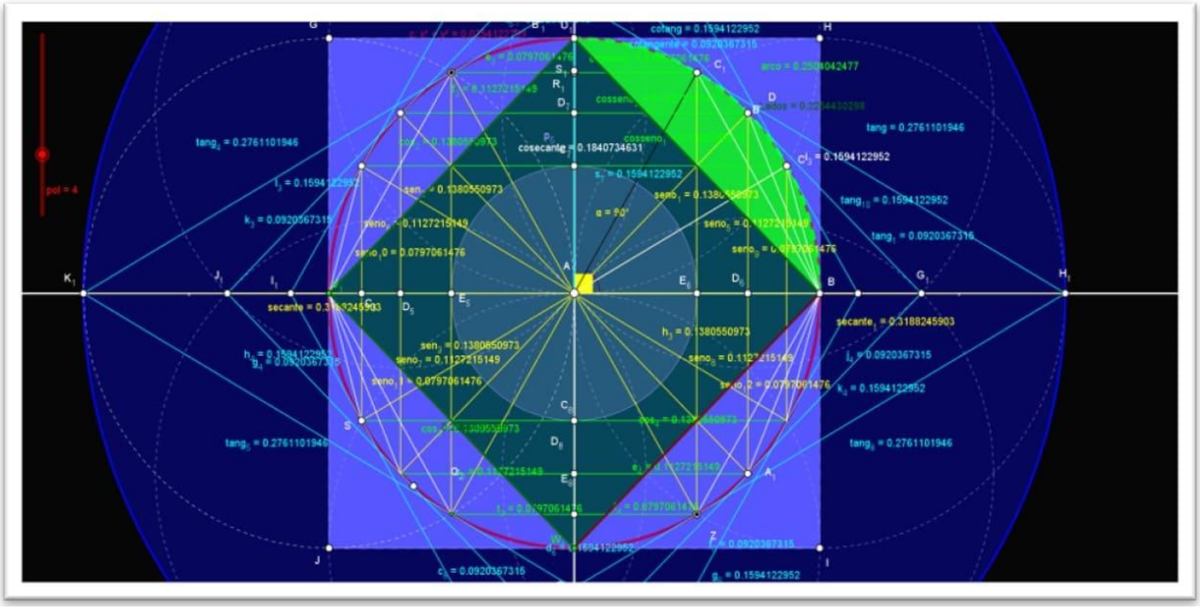
Figure 33. Demonstration of rational division proving with calculations (0.9999...≠1)



circulo completo infinito.ggb

Arquivo Editar Exibir Opções Ferramentas Janela Ajuda

	A	B	C	D	E	F	G	H	I	J	K
1	Circunferê...	1.00042	2.00084	3.00126	4.00168	5.0021	6.00252	7.00294	8.00336	9.00379	10.00421
2	Diâmetros	0.31844	0.63689	0.95533	1.27378	1.59222	1.91066	2.22911	2.54755	2.86599	3.18444
3	Divisão(B1...	3.14159	3.14159	π	3.14159	3.14159	π	3.14159	3.14159	3.14159	3.14159
4	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante	Constante
5	Seno	0.13789	0.27578	0.41367	0.55156	0.68945	0.82734	0.96523	1.10312	1.24101	1.3789
6	Cosseno	0.07961	0.15922	0.23883	0.31844	0.39805	0.47767	0.55728	0.63689	0.7165	0.79611
7	Tangente	0.27578	0.55156	0.82734	1.10312	1.3789	1.65468	1.93046	2.20624	2.48202	2.7578
8	Cotangente	0.09193	0.18385	0.27578	0.36771	0.45963	0.55156	0.64349	0.73541	0.82734	0.91927
9	Cossecante	0.18385	0.36771	0.55156	0.73541	0.91927	1.10312	1.28698	1.47083	1.65468	1.83854
10	Secante	0.31844	0.63689	0.95533	1.27378	1.59222	1.91066	2.22911	2.54755	2.86599	3.18444
11	Radianos	0.16674	0.33347	0.50021	0.66695	0.83368	1.00042	1.16716	1.33389	1.50063	1.66737
12	Polígonos	0.15922	0.31844	0.47767	0.63689	0.79611	0.95533	1.11455	1.27378	1.433	1.59222
13	Diferença	0.00751	0.01503	0.02254	0.03006	0.03757	0.04509	0.0526	0.06012	0.06763	0.07515

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2.7 PROOFS OF THE PYTHAGOREAN THEOREM AND THE RATIONAL SQUARE ROOT $\sqrt{2}$

The Pythagorean theorem is named after the Greek mathematician Pythagoras (570 B.C.-495 B.C.). The theorem's definition is a mathematical relationship between the lengths of the sides of a right triangle. The area of the square whose side is the hypotenuse is the sum of the areas of the squares whose sides are the catheti $a^2 + b^2 = c^2$.

2.8 THE DIAGONAL (D) OF THE SQUARE

Pythagoras knew at the time, however, that his theorem had a flaw. When the legs of the triangle were equal, his theorem did not work because there would be no irrational measure for the hypotenuse $\sqrt{2}$ Mathematicians who succeeded him since that time tried to understand why the sides of the triangle did not have a common measure, they could not be measured exactly through a unit common to both.

Indeed, suppose the hypotenuse is (p), and the legs are equal and represented by the letter (q). It is known that p / q is an irreducible fraction, leaving an even numerator and an odd denominator, or vice versa. The application of the theorem results in ($p^2 = 2q^2$). Obviously, (p^2) is even, as it is twice (q^2), that is, it comes from a product of a number multiplied by 2. If p^2 is even, it implies that p is even (the square of an even number is always even, and the square of an odd number is always odd), so (q) must be odd, otherwise the given fraction would not be irreducible. Now, let's make (p = 2k), since p is even, we rewrite: $(2k)^2 = (2q^2)$, simplifying results in ($4k^2 = 2q^2$) which results in ($2k^2 = q^2$).

Let's calculate the diagonal (d) of the square as a function of the side L. The problem can also be formulated as follows: given the side L, calculate the diagonal (d). Applying the Pythagorean theorem, we can calculate the hypotenuse from the squares of the legs $d^2 = 1^2 + 1^2 \leftrightarrow d = \sqrt{2}$.

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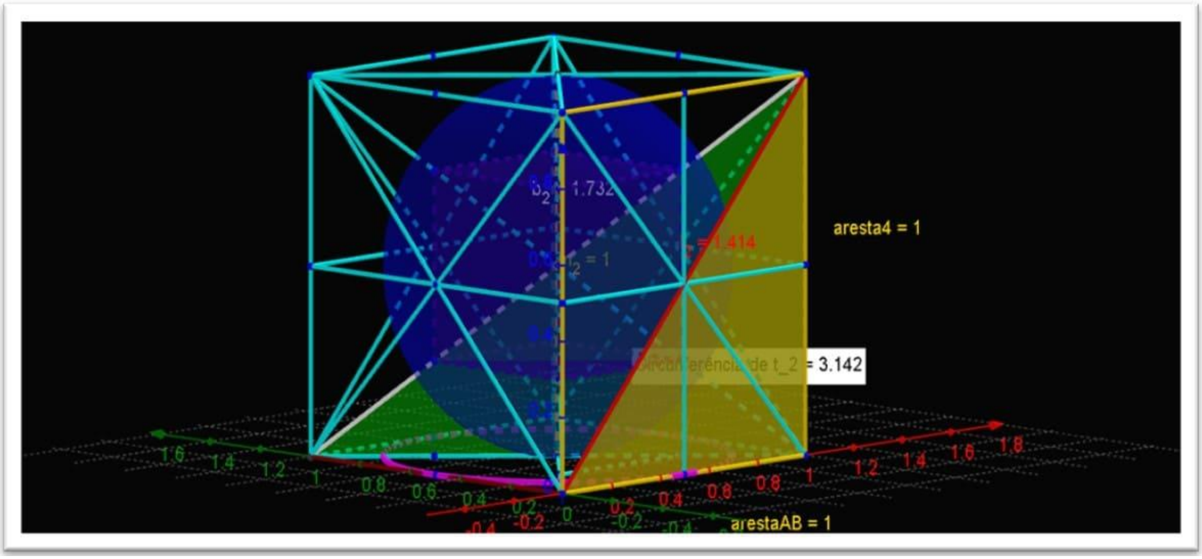
Figure 34. Demonstrations of rational results of the Pythagorean theorem



$\frac{p}{q} = 2.$	$\frac{1.414\,213\,562\,372\,821\,413\,772\,808\,569\,450\ldots}{0.707\,106\,781\,185\,750\,093\,488\,541\,834\,057\ldots} = 2.$		
$a^2 = b^2 + c^2$	(1.414 213 562...) 2	(1)2	(1)2
Resultados	2	1	1
$\frac{p}{q} = 2.$	$\frac{2.828\,427\,124\,745\,642\,827\,545\,617\,138\,900\ldots}{1.414\,213\,562\,372\,821\,413\,772\,808\,569\,450\ldots} = 2.$		
$a^2 = b^2 + c^2$	(2.828 427 125...) 2	(2)2	(2)2
Resultados	8	4	4
$\frac{p}{q} = 2.$	$\frac{4.242\,640\,687\,119\,144\,301\,359\,040\,778\,953\ldots}{2.121\,320\,343\,559\,572\,150\,679\,520\,389\,476\ldots} = 2.$		
$a^2 = b^2 + c^2$	(4.242 640 687...) 2	(3)2	(3)2
Resultados	18	9	9
$\frac{p}{q} = 2.$	$\frac{5.656\,854\,249\,491\,986\,986\,576\,020\,191\,374\ldots}{2.828\,427\,124\,745\,642\,827\,545\,617\,138\,900\ldots} = 2.$		
$a^2 = b^2 + c^2$	(5.656 854 249...) 2	(4)2	(4)2
Resultados	32	16	16
$\frac{p}{q} = 2.$	$\frac{7.071\,067\,811\,865\,709\,985\,751\,819\,284\,967\ldots}{3.535\,533\,905\,931\,305\,254\,598\,560\,509\,792\ldots} = 2$		
$a^2 = b^2 + c^2$	(7.071 067 812...) 2	(5)2	(5)2
Resultados	50	25	25

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$\frac{p}{q} = 2.$	$\frac{8.485\,281\,374\,238\,288\,602\,718\,081\,557\,907\ldots}{4.242\,640\,687\,119\,144\,301\,359\,040\,778\,953\ldots} = 2$		
$a^2 = b^2 + c^2$	(8.485 281 374...) 2	(6)2	(6)2
Resultados	72	36	36
$\frac{p}{q} = 2.$	$\frac{9.899\,494\,936\,611\,993\,980\,052\,546\,998\,954\ldots}{4.949\,747\,468\,305\,996\,990\,026\,273\,499\,477\ldots} = 2\,.$		
$a^2 = b^2 + c^2$	(9.899 494 937...) 2	(7)2	(7)2
Resultados	98	49	49
$\frac{p}{q} = 2.$	$\frac{11.313\,708\,498\,984\,581\,013\,073\,662\,232\,124\ldots}{5.656\,854\,249\,491\,986\,986\,576\,020\,191\,374\ldots} = 2\,.$		
$a^2 = b^2 + c^2$	(11.313 708 5...) 2	(8)2	(8)2
Resultados	128	64	64
$\frac{p}{q} = 2.$	$\frac{12.727\,922\,061\,357\,432\,904\,077\,122\,336\,860\ldots}{6.363\,961\,030\,678\,716\,452\,038\,561\,168\,430\ldots} = 2\,.$		
$a^2 = b^2 + c^2$	(12.727 922 06...) 2	(9)2	(9)2
Resultados	162	81	81
$\frac{p}{q} = 2.$	$\frac{14.142\,135\,623\,731\,419\,971\,503\,638\,569\,934\ldots}{7.071\,067\,811\,865\,709\,985\,751\,819\,284\,967\ldots} = 2\,.$		
$a^2 = b^2 + c^2$	(14.14213562...)2	(10)2	(10)2
Resultados	200	100	100

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2.9 THE DIAGONAL (D) OF THE CUBE

Let's calculate the diagonal (D) of the cube as a function of the side L. Applying the Pythagorean theorem to the triangle, we have $d=a\sqrt{3}=1{,}732...$

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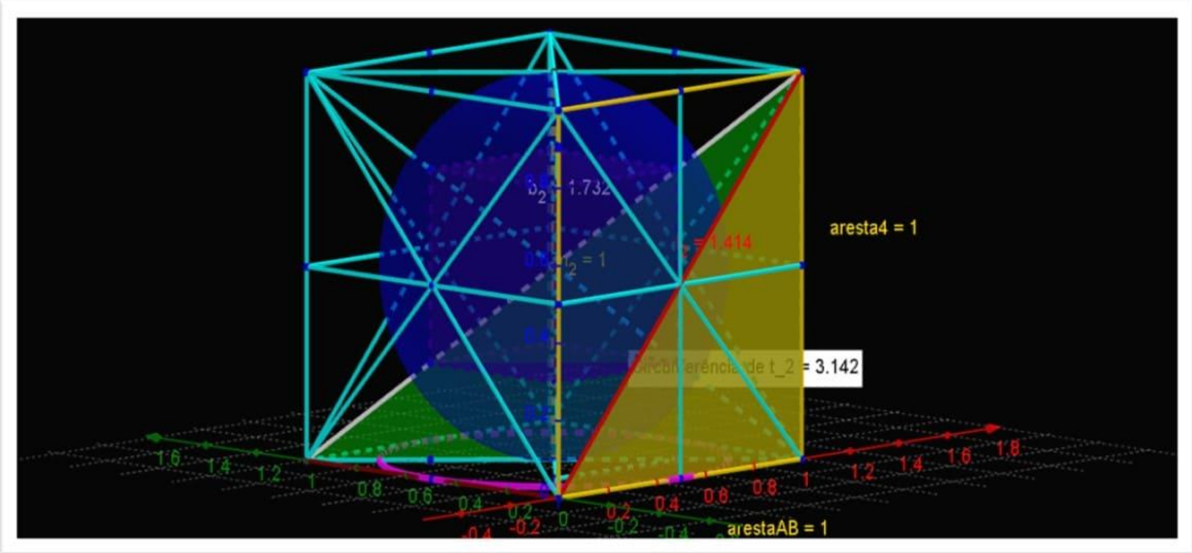
Figure 35. Demonstrations of the results of the Pythagorean theorem: ($p^2 = 2q^2$)



$\frac{p}{q} = 2$	$\frac{1.7320508075697824935941592l901}{0.8660254037848912467970796095O5} = 2.$		
$a^2 = b^2 + c^2$	(1.732050807)2	(1.414213562372)2	(1)2
resultados	3	2	1

$\frac{p}{q} = 2$	$\frac{3.4641016151377884567936381407 88}{1.7320508075697824935941592190 1} = 2.$		
$a^2 = b^2 + c^2$	(23.464101615)	(2.82842712474)2	(2)2
resultas	12	8	4

$\frac{p}{q} = 2$	$\frac{5.1961524227068268519045721118 3}{2.5980762113525664597219395921 3} = 2.$		
$a^2 = b^2 + c^2$	(25.1961524227)	(4.24264068711)2	(3)2
resultados	27	18	9

$\frac{p}{q} = 2$	$\frac{6.9282032302757691358727628157 7}{3.4641016151377884567936381407 88} = 2.$		
$a^2 = b^2 + c^2$	(26.9282032302)	(5.656854249491)2	(4)2
resultados	48	32	16

$\frac{p}{q} = 2$	$\frac{8.6602540378447114198409535197 2}{4.3301270189223557099204767598 6} = 2.$		
$a^2 = b^2 + c^2$	(28.660254037)	(7.07106781186)2	(5)2
resultados	75	50	25

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$\frac{p}{q} = 2$	$\frac{10.392304845413653703809144223 7}{5.196152422706826851904572111 83}$ = 2.		
$a^2 = b^2 + c^2$	(210.392304845)	()28.4852813742	(6)2
resultados	108	72	36

$\frac{p}{q} = 2$	$\frac{12.1243556529825959877773349276}{6.0621778264912979938886674638 0}$ = 2.		
$a^2 = b^2 + c^2$	(212.12435565)	(9.8994949366)2	(7)2
resultados	147	98	49

$\frac{p}{q} = 2$	$\frac{13.8564064605515382717455256315}{6.9282032302757691358727628157 7}$ = 2.		
$a^2 = b^2 + c^2$	(213.85640646)	(11.3137084989)2	(8)2
resultados	192	128	64

$\frac{p}{q} = 2$	$\frac{15.5884572681204805557137163355}{7.7942286340602402778568581677 4}$ = 2.		
$a^2 = b^2 + c^2$	(215.58845726)	(12.7279220613)2	(9)2
resultados	243	162	81

$\frac{p}{q} = 2$	$\frac{17.3205080756894228396819070394}{8.6602540378447114198409535197 2}$ = 2.		
$a^2 = b^2 + c^2$	(217.320508075)	(14.14213562373)2	(10)2
resultados	300	200	100

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Figure 36. Trigonometric circle at (3 d) of atomic rational values

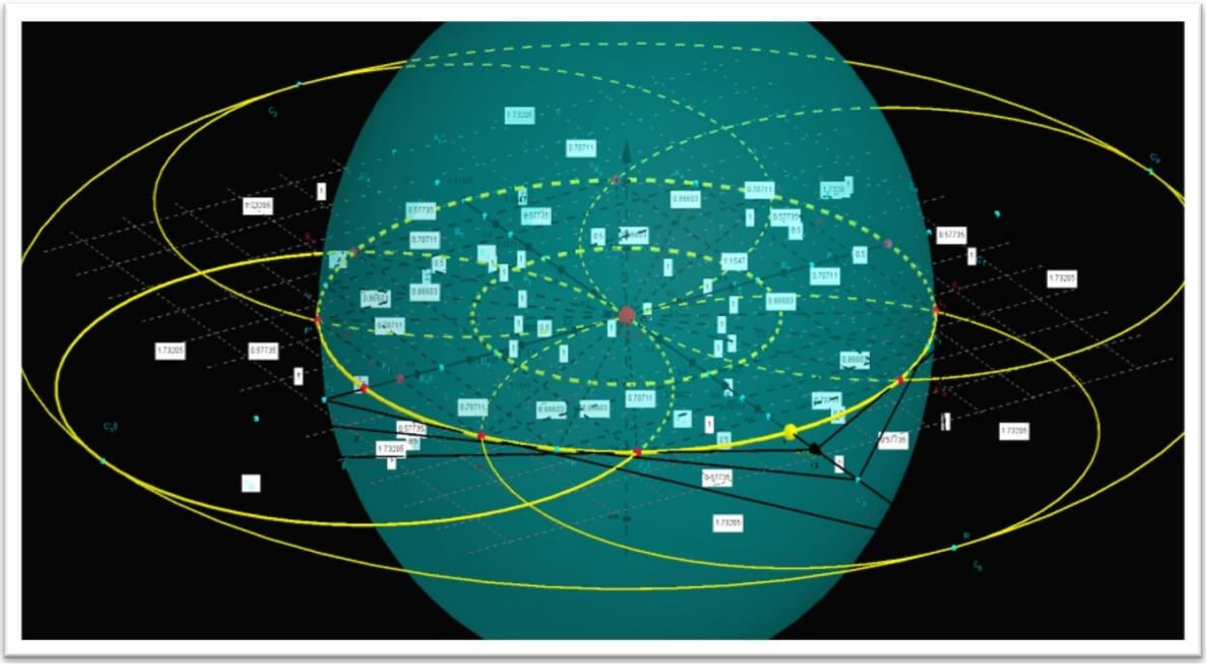

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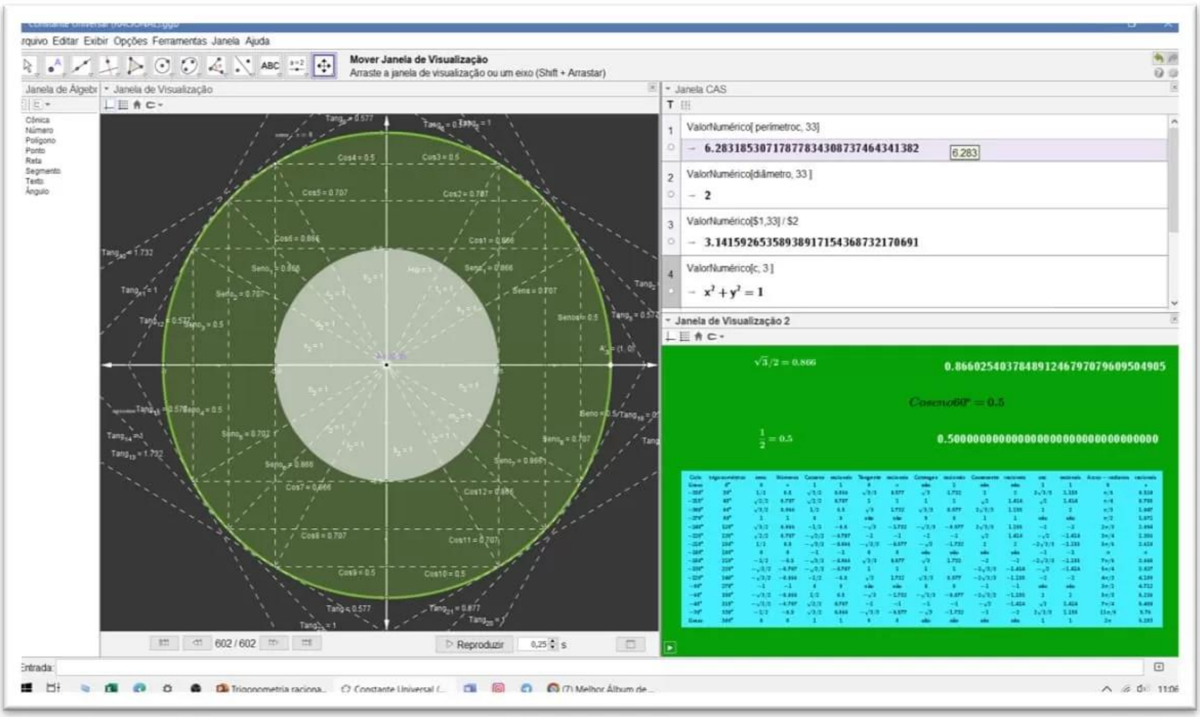
Figure 37. Demonstration of trigonometric circle in (3 d) of rational values



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3. FINAL CONSIDERATIONS

Mathematicians from various times sought to find a rationality for the constant π. However, they came to an incredible discovery for the time of the existence of irrational numbers. The proof that the constant π is irrational was made by Johann Lambert in 1761 and Legendre in 1794. In addition to being irrational, π is a transcendental number, which was proved by Ferdinand Lindemann in 1882. This means that there is no polynomial with integer or rational coefficients. The decimal representation is unpredictable.

The results of the demonstrations of the Cartesian, isometric, and polar mathematical models, with infinite periodic rational calculations, can be applied in all sciences, geometries of circular and spherical bodies, physics, and astronomy.

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